APPLIED ANCIENT ASTRONOMY

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Data from ancient astronomy can be applied to a study of the effects that govern the long-term evolution of the solar system. This field of application is now growing almost explosively. Many results obtained during the past few years are negative in the sense that they show "well-established" results of a few years ago to be either unfounded or wrong, and at present we have little understanding of the mechanisms involved. However, we have reason to hope that our understanding will grow rapidly in the next few years.

AROUND 1950 IT WAS FASHIONABLE IN MANY scientific circles to talk about the difference between basic and applied research. Most of these discussions never yielded a useful definition of the difference, perhaps because the discussants usually based their approach upon the subject matter of the research. It may be more fruitful to base the definition upon the motivation of the research worker: If the goal of the worker is to understand something simply for the sake of understanding, the research is pure. If the goal is to use material from one field as an aid to understanding a different field, the research is applied.

In this sense, the research that I have done in ancient (and medieval) astronomy is applied, even though the subject is usually considered to be among the purest of the pure. The manner of my involvement in ancient astronomy illustrates the peculiar turns and the unexpected shifts in direction that often arise in the pursuit of a research goal.

Celestial navigation is based upon observing the sun, moon, planets, and stars. In order to find out where he is, the navigator must use a position of, say, the sun at the time he observes it, and the position that he uses is necessarily based upon prediction. An American navigator takes the position of the sun from the *American Ephemeris and Nautical Almanac*[1], sometimes through the intermediary of a navigator's handbook. Navigation by means of the Navy Navigation Satellite System (Transit) is based upon observing a satellite in the system, and the navigator must use the position of the satellite at the time of the observation.

This position, like a position used in celestial navigation, must be a predicted one.

Provision of a predicted position to the navigator has always been one of the most difficult problems in the Transit System. When the system was first devised[2], we did not think that it would be possible to predict the position with enough accuracy for more than about half of a day, and the satellite memory, the configuration of the ground support system, and the computing procedures were all designed to cope with this limitation.

After we completed the teething stage of the system, some of those in the program turned their attention to the possibilities of predicting satellite position for a longer interval. Many things are needed in order to predict position for a long time, but a minimum is clearly an accurate knowledge of the force system acting on a satellite. About 1964, as a part of studying the force system, I calculated the perturbation acting on one of the navigation satellites as a result of the sun's gravitation, and I subtracted this perturbation from the observed ephemeris of the satellite. When I did so, I was surprised to find that the position still contained a perturbation with the same time dependence, but merely reduced in amplitude. After some consideration, I found[3] the source of this remaining perturbation: Solar gravitation distorts the mass distribution of both the solid earth and the oceans, producing what is called the solar tide. The perturbation in question arises from the gravitation of the mass that participates in the solar tide.

The solar tide is a complex function of time with a complicated power spectrum. For definiteness here, we may concentrate our attention on the component whose period is half of a solar day; this component can be described by means of an amplitude and a phase angle. The obvious way to account for the tidal perturbation upon the satellite motion is then to find the amplitude and phase angle from measurements of the tides. In order to explain why the obvious procedure does not work, it is necessary to describe the tides in a little more detail.

The Tide-Raising Force and Response

Tides caused by the moon are actually about twice as large as those caused by the sun, but their effects on satellites are considerably less. The reason is concerned with the frequency of revolution of the sun or moon. The tidal distortion due to a lunar tide revolves with the moon and that due to the sun revolves with the sun. The effect of either tide depends upon the frequency of revolution of the sun or moon with respect to the satellite orbit. Thus the frequency of the lunar tide is about 13 times the frequency of the solar tide, so far as the dynamics of satellites are concerned. As a result, the perturbation due to the lunar tide is $13/2$ or $\approx 1/6$ of that due to the solar tide.

For this reason, much of the discussion of this paper will be in terms of the solar tide. Most of the discussion of the tide-raising force and response will apply to the moon as well as to the sun, at least qualitatively.

Let us focus attention upon the points marked 0, 1, and 2 in Fig. 1; point 0 is the center of the earth. The sun's gravitation attracts all parts of the earth toward it, and it is this attraction that makes the earth orbit about the sun. However, the sun's gravitation is slightly greater at point 1 than at point 0, and hence the sun tends to pull point 1 away from the center. Similarly, the sun's gravitation is greater at point 0 than at point 2, and the sun tends to pull the center of the earth away from point 2. If we refer the effects to the center 0, then, the sun tends to pull both points 1 and 2 away from the center, as is suggested by the ellipse in Fig. 1.

As the earth rotates with respect to the sun, each part of it is urged up and down twice a day by the tide-raising force. In other words, we have a physical system, the earth, that is subject to a

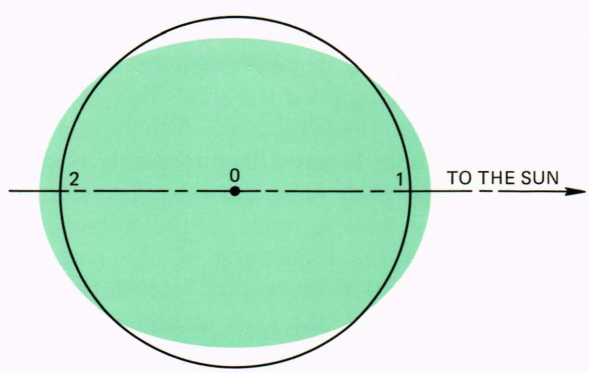

Fig. 1—The origin of the solar tide. The sun's gravitation pulls point 1 away from the center 0, and it pulls the center 0 away from the opposite point 2. Since the distortion takes place at constant volume, the points along the vertical in the figure move toward the center, and the resulting figure of the earth is an ellipsoid whose long axis points at the sun.

periodic perturbing force with a period of 12 hours. The nature of the response depends upon the physical characteristics of the system being forced. Actually, the earth contains several components that have different characteristics and hence different responses.[4]

First, there is the liquid core. Because it is close to the center, the tide-raising force on it is small and we can neglect its response.

Second, there is the solid part of the earth, whose characteristic periods are considerably shorter than 12 hours. Theories of the earth indicate that the response should be in phase with the disturbing force and that the amplitude should be of the order of 25 cm at the equator. Direct measurements of the body tide, as it is called, give results that are approximately in agreement.

Third, there is the ocean. The ocean consists of a number of basins that interact relatively little with each other and have different dynamic characteristics. Thus it is more appropriate to speak of the oceans when dealing with the tides. In some oceans, the response is in phase with the tide-raising force (direct tide), and in others, the response is out of phase (inverted tide). From the standpoint of the gravitational potential due to the tides in all the oceans, we can speak of a resultant ocean response, but it is a matter of great delicacy to decide whether the resultant is direct or inverted, and what its amplitude is.

In order to find the resultant ocean tide, then, we need careful measurements made on an ocean-wide basis, but this is just what we did not have ten years ago. Until recently, it was not possible to measure the ocean tide except at the shore, where conditions are greatly distorted by the presence of the rigid crust. The response in the middle of the ocean cannot be inferred with confidence from measurements made at the distorting edges. Thus it is not possible to calculate the tidal perturbation on a satellite solely from external evidence.[a]

The continual motion of the earth and ocean in response to the tide-raising force must be accompanied by friction. The first-order effect of friction in a vibrating system is a shift in phase between the perturbation and the response, without a change in amplitude. From the properties of the materials involved, we may speculate that friction in the ocean tide is considerably more important than friction in the body tide.

Direct measurements of the tide cannot give us the phase of the total tide, for the same reason that they cannot give us the amplitude.

Tidal Parameters from Satellite Tracking Data

Since it was not possible to take values of the amplitude and phase from other data and to use them in calculating the tidal perturbation on a satellite, it was necessary to find the tidal parameters by analysis of the measured perturbation. Kozai[5] and I[3] have done this, using different satellites and different methods of analysis. The results agree as well as can be expected.

The amplitude of response that is found from the satellite data is about equal to that which is found for the solid earth. In other words, the resultant response of all the ocean tides is nearly zero. However, anyone who has visited the seashore is well aware that the response at most particular places is far from zero. Thus it must be that direct and inverted tides in the oceans have about equal importance.

At present there is no independent method of measuring the total tide parameters, and there is no assured way of assessing the probable error in this result. We can only try to assess the level of error that is consistent with the data and the general situation. With regard to the amplitude, we can speak with fair assurance; it is not likely that the inferred amplitude is in error by more than about 10 %.

The situation is quite different with regard to the phase angle. Kozai's method is not particularly sensitive to the phase, so I shall discuss only my own results. The inference of the tidal phase from the satellite is sensitive to the spatial orientation (position of the node) of the satellite orbit. When an inference procedure is sensitive to the value of a parameter, it is highly desirable to use data in which the parameter is well distributed over its possible range of values. By an unfortunate accident, the only orbits that were available at the time of my work had their nodes distributed only over about half of a circle, and there is thus a possibility of a serious bias.

The phase angle obtained from the satellite data is about 2°. Specifically, this means that the longi-

[a] Some techniques of measuring the tides in the open ocean have been developed within the past few years, and at some time it will be possible to find the ocean tide by direct measurement.

tude of the place where the tide is high is 2° east of the longitude of the sub-solar (or sub-lunar) point, on the average. I estimate the maximum possible effect of the bias just mentioned as a factor of two. If this is correct, the phase angle is between 1° and 4°.

The parameters found for the solar and lunar tides are approximately equal.

Tidal Friction and Other Non-Gravitational Effects in Astronomy

The phase angle of the tides has important consequences in astronomy that have been studied intensively for about three centuries. Thus it occurred to me that these astronomical studies might supply a partial check on the satellite results.

Consider friction in the lunar tide first. This friction converts mechanical energy in the earth-moon system into heat, but since it is internal to the system it does not change the total angular momentum. Since the earth's rotation contains most of the rotational energy of the system, the effect is to take angular momentum from the earth's spin and transfer it into orbital angular momentum of the moon. Since the earth is reasonably rigid, it can decrease its angular momentum only by decreasing its spin rate. The moon is a satellite, however, and it can increase its angular momentum only by moving into a larger orbit and consequently decreasing its orbital angular velocity.

In other words, friction in the lunar tide tends to give a negative acceleration both to the earth's spin and to the moon's orbital angular velocity. So far as we know, this is the only source of a lunar acceleration that is large enough to be measurable. There are several other sources of an acceleration of the earth's spin, however.

There is friction in the solar tide. This friction is internal to the system consisting of the earth and sun; it takes energy from this system without changing its angular momentum. It does this by transferring angular momentum from the earth's spin to the earth's orbital motion around the sun. The effect on the orbital motion is about 7 or 8 orders of magnitude too small to be measured with present techniques. Thus, for present purposes, friction in the solar tide gives a negative acceleration to the earth's spin without affecting the orbital motion of the earth, and also without affecting the orbital motion of the moon.

There are many other possible sources of a spin acceleration, several of which are discussed by Jeffreys and by Munk and MacDonald.[4] Many of them cannot be evaluated quantitatively in our present state of knowledge. Many, but by no means all, arise from processes that are internal to the earth. These can change the spin only by acting on the moment of inertia; an example is a possible change in the mean temperature of the earth.

In summary, there are a number of non-gravitational effects that are changing the spin rate of the earth and the orbital motion of the moon. We are not able to evaluate the resultant of these effects quantitatively from evidence that is external to astronomy, and hence we must attempt to estimate the resultant on the basis of astronomical data. In particular, if we can estimate the acceleration of the moon from astronomical data, we shall have a test of the tidal phase that was inferred from satellite data.

Solar Time and Dynamical Time; Some Notation

Let I denote the moment of inertia of a body that is in angular motion, let θ denote an angular coordinate, and let T denote the torque that is acting on the body. A common form of the equation of motion is then

$$T = \frac{d}{dt}\left(I\frac{d\theta}{dt}\right). \tag{1}$$

Most texts on mechanics point out that Eq. (1) is valid only if we impose certain restrictions on the positional coordinate system in which we measure θ. Fewer texts point out that there are also certain restrictions on the time base. (See Table 1 for the principal nomenclature in this discussion.)

As an example of what we mean by time bases, let us suppose that the "big bang" theory is correct, and let 0 be the epoch of the bang. Suppose that t is a time base that is 0 at the bang and that increases monotonically thereafter. Let

$$s = ln\ (t + 1). \tag{2}$$

Then both s and t are 0 at the epoch of the bang, and both increase inexorably toward infinity thereafter. Is there any property that makes t, say, a more useful time base than s?

The concept of time and of its measurement poses deep philosophical problems that can be answered only with difficulty if at all. I shall attempt to short-circuit them by use of Eq. (1). Let us measure T, I, and θ in Eq. (1). Instead of regarding Eq. (1) as a law of physics, we adopt it as the *definition* of time t. A time base defined by means of Eq. (1), or of an analogous equation, can be called dynamical time. This definition by no means answers all problems connected with defining time; for example, it presupposes that we have a way of defining the torque T that is independent of Eq. (1). In the physical sciences, a time base that is dynamical is more useful than one that is not.

The time base by which we lead our daily lives, and by which I believe all peoples have led their daily lives, is solar time. Solar time is just the angle between some fixed direction in the earth, such as our local earth radius, and the line from the center of the earth to the center of the sun, projected onto the plane of the equator.[b] Within the present century, it has been realized that solar time is not a dynamical time base and that it fails at a level of accuracy of the order of 10^{-7} or 10^{-8}.

It is now useful to introduce some notation. In the rest of this paper, τ will denote solar time. t will denote a dynamical time base that is related to τ by

$$t_0 = \tau_0, \ (dt/d\tau)_0 = 1, \qquad (3)$$

in which the subscript 0 refers to an epoch that is close to the present; it is not necessary to specify this epoch rigorously in this paper. A prime will denote a derivative with respect to τ and a dot will denote a derivative with respect to t.

Let θ denote the angular position of some body in the solar system; a particular body will be identified by means of an appropriate subscript. ν will denote the corresponding angular velocity with respect to τ, and n will denote the angular velocity with respect to t:

$$n = \dot{\theta} = d\theta/dt, \ \nu = \theta' = d\theta/d\tau = n(dt/d\tau). \quad (4)$$

We also need the accelerations $\dot{n}$ and ν' with respect to t and τ, respectively, which are related by

$$\nu' = \dot{n}(dt/d\tau)^2 - n(dt/d\tau)^3 \, (d^2\tau/dt^2). \quad (5)$$

We have been indoctrinated to think that the Copernican, or heliocentric, picture of the solar system is the correct one and that the ancient geocentric picture is wrong. Actually, the difference between the pictures, if it has any importance at all, is important for philosophy; it is a mere matter of convenience so far as the physical sciences are concerned. For present purposes the geocentric picture is more convenient. In it, so far as major effects are concerned, we can represent the position of the sun by a single coordinate θ_S, called the mean longitude of the sun; we can represent the position of the moon by a coordinate θ_M, called the mean longitude of the moon, and we can represent the spin orientation of the earth by a coordinate θ_e, which will be called the angular position of the Greenwich meridian.

In the literature in this field, it is customary to use the following units:

An angle is always measured by seconds of arc;

Time is always measured by means of the Julian century, which means 100 Julian years, or exactly 36525 days.

These units are so well understood in the literature that they are usually omitted, and explicit statements of them will be omitted in the rest of this paper.

We want to apply Eq. (5) to any of the angles θ_S, θ_M, or θ_e. In doing so, we remember that solar time τ, except for units, is equal to $\theta_e - \theta_S$. This fact allows us to calculate $d^2\tau/dt^2$. Further, since $d\tau/dt = 1$ at an epoch close to the present (Eq. (3)), we can take $d\tau/dt$ as unity with enough accuracy for the historical period, although not for geological time. This gives us

$$\nu' = \dot{n} - n\frac{\dot{n}_e - \dot{n}_S}{n_e - n_S} \qquad (6)$$

for the relation between ν', an acceleration with respect to solar time, and $\dot{n}$, an acceleration with respect to dynamical time. In using Eq. (6), we can take $\dot{n}_S = 0$ to high accuracy, and this approximation is usually made.

The Fundamental Problem in the Use of Ancient Data

The specific problem that led us to consider the use of astronomical data was, as we said above, the estimation of the amount of tidal friction. From a less narrow point of view, we want to study non-gravitational effects in the solar system, but, in doing so, we want to separate tidal friction from other effects. I shall use the term "non-fric-

[b] Strictly speaking, what I have just defined is called apparent solar time, but most of the discussion will tacitly relate to mean solar time. Since apparent time can be converted to mean time without the use of Eq. (1), I shall ignore the distinction in order to save space.

tional effects" to mean the totality of all non-gravitational effects other than tidal friction.

The best information we have says that the acceleration of the moon results only from tida friction, at the level of accuracy that concerns us here. If this be so, we can calculate the amount of tidal friction from an estimate of the lunar acceleration. However, the acceleration we need in order to calculate tidal friction is $\dot{n}_M$, the acceleration with respect to dynamical time, rather than ν_M', the acceleration with respect to solar time.

It is safe to assume that atomic time provides a much better embodiment of a dynamical time base than does solar time. In consequence, studies of the position of the moon as a function of atomic time give us an estimate of $\dot{n}_M$, and there have been three such estimates.[6] The results scatter somewhat, but it is probably fair to represent them by

$$\dot{n}_M = -40 \pm 10. \tag{7}$$

However, estimating the value of $\dot{n}_M$ in this way is just on the verge of the possible. The time interval available for the estimate is only about 15 years (0.15 century), and the measurable effect on the position of the moon is about 0″ .05. Further, the data samples used in the three studies have considerable interdependence. These conditions favor the possibility of experimental errors that are common to all three studies, and it is conceivable that the true error in Eq. (7) is greater than that given.

The phase angle of the tides required to produce $\dot{n}_M = -40$ is about 4°, whereas the value from satellite data that is quoted above is about 2°. As we shall see below, the larger value has independent confirmation and, in the present state of knowledge, it is more likely to be correct.

At the time I began my studies of non-gravitational effects, the data with an atomic time base were not available, and it was therefore necessary to work with solar time. The lunar acceleration that we derive from the astronomical data is, in consequence, ν_M' rather than $\dot{n}_M$. Now, if we apply Eq. (6) to the moon, inserting the known values of n_e and n_S, and make the assumption $\dot{n}_S = 0$, we find

$$\nu_M' = \dot{n}_M - 0.036\,601\,\dot{n}_e, \tag{8}$$

in the system of units that has been adopted. Thus we cannot get $\dot{n}_M$ from ν_M' unless we know $\dot{n}_e$.

In order to get $\dot{n}_e$, we apply Eq. (6) to the sun, getting

$$\nu_S' = -0.002\,738\,\dot{n}_e. \tag{9}$$

It looks, therefore, as if the problem is simple: We measure ν_S' and ν_M' by means of astronomical data and solve Eqs. (8) and (9) simultaneously for $\dot{n}_M$ and $\dot{n}_e$. These values in turn can be used to give us the frictional and the non-frictional parts of the non-gravitational effects. The difficulty arises from the nature of the data available.

Data from modern times are of course the most accurate, but they are affected only slightly by the accelerations. Thus we find it hard to estimate the accelerations from modern[c] data. As we go back in time, the effect of the accelerations goes up as the square of the age of the data, but the accuracy of the observations goes down.

It turns out that squaring the time probably wins out over the decreased accuracy, at least back to classical Greek times, but this victory is won at a price. The price is that most of the data relate to the moon and not to the sun. Thus we have highly detailed knowledge of ν_M', but we have relatively poor knowledge of ν_S' and hence of the accelerations with respect to dynamical time. I should qualify this by pointing out that the statement rests upon the data that have been most studied. These data come primarily from the Greek, Islamic, and European cultures. The situation may be altered if and when more data from China, India, and possibly other areas become available.

It is interesting to look briefly at two examples of ancient data. One example gives an estimate of ν_S' and the other gives an estimate of ν_M'.

Hipparchus's Solar Data

In the years from –161 to –127 BC, the great Hellenistic astronomer Hipparchus made a series of observations of the vernal and autumnal equinoxes on the island of Rhodes. He probably made several observations of the solstices as well, but only one, of the summer solstice of –134, has survived. Except for one isolated observation of the summer solstice of –279, Hipparchus's data are the oldest solar data that have been published. I estimated ν_S' from them in Chapter II of an earlier work.[7]

After the observations by Hipparchus, it is al-

[c] Modern, in astronomical parlance, is often used to signify the period since the adoption of the telescope and the pendulum clock roughly three centuries ago.

most a thousand years before we find any more solar observations. Islamic astronomers began making systematic observations in 829, and I have analyzed many of their equinox and solstice observations.[7,8] The Islamic data are not as useful as we might at first expect, unfortunately, because they are predominantly of the autumnal equinox, and observations of an equinox are subject to severe bias. There are two possible ways of overcoming the bias.

The bias made by a single observer tends to be equal and opposite for vernal and for autumnal equinoxes. Thus we can remove most of the bias if we can use a significant body of observations of both equinoxes made by the same observer. This is what we are able to do with Hipparchus's data.

We can also remove the bias if the observer constructed a theory of the sun from his observations, provided we understand his theory and that we have the crucial parameters in it. All theories of the sun since the time of Hipparchus, including the modern theories used in preparing the great national ephemerides,[d] involve four basic parameters. In modern theories, the parameters are called the eccentricity, the mean longitude of perigee, the mean motion, and the mean longitude at the epoch. Older theories have various forms, but the parameters in them are either equivalent to these four or can be converted into them.

In several cases, we have only a single equinox observation made by a particular Islamic astronomer, but we also have his solar theory in at least partial form. We can remove the bias in his equinox observation if we have his values of the eccentricity and longitude of perigee, or their equivalents. When we have these parameters, and one or more equinox observations, we can in effect calculate the solstice observations and the remaining equinox observations that he used in formulating his theory. In a recent work,[9] I have used several Islamic solar theories in order to estimate ν_S'. Before I did so, I tested my ability to understand and to use the ancient theories by working with the theory of Hipparchus, because we have both his theory and the observations upon which he based it. It is simpler to give the analysis based upon his theory than the analysis based upon his observations.

[d] The standard American ephemeris publication is the *American Ephemeris and Nautical Almanac,* which has already been cited.[1]

Hipparchus's theory, written with the aid of modern algebraic and trigonometric notation, can be embodied in the equations:

$$\begin{aligned} \lambda_S &= \theta_S + e_S, \\ \theta_S &= 330^\circ.75 + 0^\circ.98563\,52784\,D, \\ M_S &= \theta_S - 245^\circ.5, \\ e_S &= \tan^{-1}\{2.5\sin M_S/(60 - 2.5\cos M_S)\}. \end{aligned} \qquad (10)$$

In Eqs. (10), λ_S is the longitude of the sun and D is the time in days from noon, apparent solar time at Alexandria, on –746 February 26.[e] θ_S, which is called the mean longitude of the sun, is the coordinate that was introduced earlier. Equations (10) agree quite well with Hipparchus's data.

For example, an observation upon which Hipparchus laid considerable weight was an autumnal equinox that he put at 06 hours on –145 September 27. For this epoch, Eqs. (10) yield the values

$$\theta_S = 182^\circ.173, \quad M_S = -63^\circ.327, \quad e_S = -2^\circ.173, \quad \lambda_S = 180^\circ.000.$$

Thus Eqs. (10) agree exactly with this observation, to the precision kept in the computations.

Newcomb's theory[f], which uses the fundamental epoch of Greenwich mean noon on 1899 December 31, gives 181° .963 for the value of θ_S at the epoch of Hipparchus's equinox. Hence the acceleration has changed θ_S by about 0° .210 in the 20.44 centuries between Hipparchus's observation and Newcomb's epoch, and

$$\nu_S' = 2 \times 0.210 \times 3600/(20.44)^2 = 3.6. \qquad (11)$$

On the basis of considerations that it would take too much space to explain, I estimate 0.7 as the uncertainty in this estimate.

Some Conjunctions of the Moon

At Alexandria, when three-fourths of the night between –294 December 20 and –294 December 21 had passed, the astronomer Timocharis[10] observed that the star β^1 Scorpii was touching the northern cusp of the moon. Calculation shows that the moon was several days past the third quarter.

[e] Strictly speaking, the expression for θ_S in Eq. (10) is taken from the form in which Ptolemy wrote Hipparchus's theory. It is likely that Hipparchus used a different fundamental epoch. Whether this is so or not, we know that the form in Eqs. (10) embodies Hipparchus's theory and that this theory is based upon his known equinox and solstice data.

[f] The needed parts of this theory are given in the section called "Explanation" in any recent volume of the *American Ephemeris and Nautical Almanac*[1]. It is necessary to include the phenomenon called aberration in order to put Newcomb's theory on a basis that is comparable with Hipparchus's theory, since Hipparchus was ignorant of abberation.

Figure 2 shows the most important features of the situation. The coordinates in Fig. 2 are celestial longitude and latitude. East (the direction of increasing longitude) is to the left in Fig. 2 instead of to the right as we see it on maps. The reason is that we look at the earth from the outside but we look at the heavens from the inside. A small circle indicates the position of the star β^1 Scorpii.

Figure 2 shows two positions of the moon; the visible disc is bounded by solid circles while the dashed line outlines the invisible part. The position to the right, where the longitude of the center is 210° .275, is the position that the moon would have had if ν_M' were identically zero and if Timocharis's measurement of the time were accurate. The other position, which is about 1° farther east, is the position indicated by Timocharis's observation.

If we calculate the lunar position as a function of ν_M' for the time that was stated, we need the value $\nu_M' = 17.3$ in order to make the calculated position agree with the stated position.

Ptolemy[10] quotes altogether eight observations of this sort, in which the position of the moon is given by referring to the star background. The earliest is the one in Fig. 2, and the latest was made on 98 January 14. It is interesting that three of the observations were made in Rome. When we

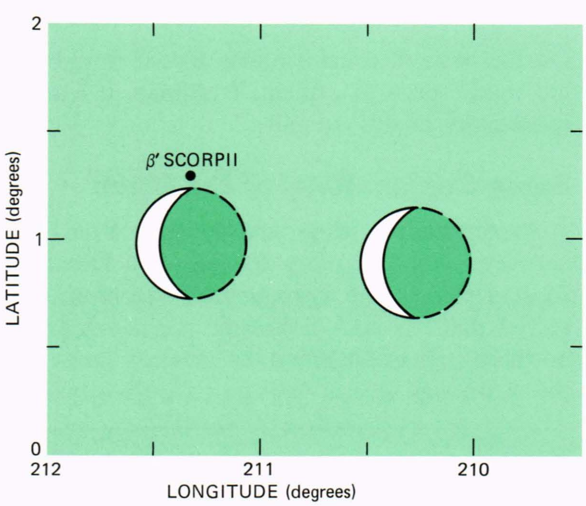

Fig. 2—Part of the sky half way between midnight and sunrise on −294 December 21 at Alexandria. According to an observation recorded by Timocharis, the star β^1 Scorpii was nearly touching the northern cusp of the waning moon. However, the position of the moon would be nearly 1° west (to the right) if the moon were not being decelerated.

analyze all eight observations, we obtain the estimate

$$\nu_M' = 8.7 \pm 1.9. \qquad (12)$$

The discrepancy between the value in Eq. (12) and the value found from Fig. 2 probably means that Timocharis made an error of about 30 minutes in his statement of the time.

The Accelerations at Two Epochs

We have two clusters of data from which we may estimate the accelerations of the sun and moon. Besides the data that have just been discussed, there are positions of planets, the magnitudes and times of both solar and lunar eclipses, and places where solar eclipses were total or nearly total. The last type of data has received more publicity than the other types, and there is a widespread tendency to consider it as the most valuable, or even the only valuable, type. Actually, it is no more valuable than several other types.

The data from around the year zero yield the estimates[g]

$$\begin{aligned} \nu_M' &= 6.5 \pm 1.9, & \nu_S' &= 3.6 \pm 0.4, \\ \dot{n}_M &= -41.6 \pm 5.9, & \dot{n}_e &= -1315 \pm 160. \end{aligned} \qquad (13)$$

The other cluster of data, which comes from the first two centuries of Islamic astronomy (say at the year 1000), with some admixture from Chinese and European sources, gives results that do not differ significantly.

The value of $\dot{n}_M$ at 0 and at 1000 is rather close to the estimate given in Eq. (7), which applies at the present time. Also, $\dot{n}_e$ is approximately the same at 0 and 1000, but we have no value for the present with which we can compare it. If we had no other information, these facts would lead us to suspect that the accelerations have been nearly constant for at least 2000 years. However, we do have a large body of data that says, with high statistical confidence, that at least one of the accelerations has changed by a large factor at two different times within this period.

The Second Derivative of the Lunar Elongation

The occurrence of eclipses, whether lunar or solar, depends upon the angle D between the moon and the sun. This angle is called the lunar

[g] Chapter XIV of a work already cited[7].

elongation. With a few trivial exceptions that I shall not take the time to describe, all old observations of eclipses give us information about D, and in particular they give us estimates of D'', the second derivative of D with respect to solar time. We see from the definition of D and from Eqs. (8) and (9) that

$$\begin{aligned} D'' &= \nu_M' - \nu_S' \\ &= \dot{n}_M - 0.033\,863\,\dot{n}_e. \end{aligned} \qquad (14)$$

We can evaluate $\dot{n}_M$ and $\dot{n}_e$ only at epochs for which we have solar data, and those epochs are approximately 0 and 1000, as we have just seen. However, data from which we can evaluate D'' are well distributed in time. I have analyzed[7,11] more than 400 observations that give estimates of D'', with dates ranging from –762 June 15 to 1288 April 2. The –8th century, which is more than a century before the Exile of the Jews in Babylon, is represented by five individual observations, and there is at least one observation in each century since then except the +3rd. When the full corpus of Chinese data becomes available, this lacuna may be removed.

Martin[12] has analyzed a body of lunar observations made after the invention of the telescope but before 1860. We can also estimate a value of D'' from Martin's work.

I have divided the total body of observations into about 25 groups and have formed an estimate of D'' from each group. The observations in each group are of the same lunar or eclipse phenomenon and they are close together in time. The results are shown in Fig. 3, which is taken from an earlier work[11] with minor modification. The distinction between different shapes of points in the figure is related only to the order in which the groups were analyzed and has no fundamental significance.

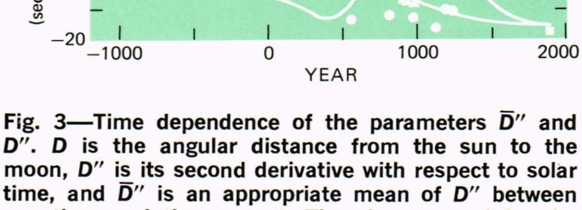

Fig. 3—Time dependence of the parameters $\bar{D}''$ and D''. D is the angular distance from the sun to the moon, D'' is its second derivative with respect to solar time, and $\bar{D}''$ is an appropriate mean of D'' between any time and the present. The shapes used for the plotted points are not significant here. The figure, with a minor change, is reproduced from *Medieval Chronicles and the Rotation of the Earth* by permission of the Johns Hopkins Press.

The curve labelled $\bar{D}''$ in Fig. 3 is the smoothest curve that I could bring myself to draw through the plotted points. I strongly suspect that the correct variation of $\bar{D}''$ is both larger and more abrupt than the curve shows. The use of $\bar{D}''$ as a symbol rather than D'' indicates a point that could not be made conveniently before: the value of an acceleration that is directly obtained from the observations is the mean value between then and now, rather than the value at the time of the observation.[h] The curve labelled D'' is derived from $\bar{D}''$ by the appropriate differentiations, and it is the best estimate that we can make of the current value of D'' at any time.

In view of the scatter of the data in Fig. 3, we cannot rely upon the curves in full detail. However, it seems safe to assume that D'' has had approximately the following behavior since –700: It was roughly constant at about −10 from about −700 to +700, it then rose fairly abruptly from −10 to about +30, where it remained until about +1200. It then fell back to −10, where it has stayed until the present.

Since D'' is a linear combination (Eq. (14)) of $\dot{n}_M$ and $\dot{n}_e$, a time variation of D'' necessarily implies a time variation of at least one of the two individual accelerations. Since the value obtained for $\dot{n}_M$ in Eqs. (13) is about the same as that in Eq. (7), it may be that $\dot{n}_M$ has remained nearly constant and that most or all of the variation has been in $\dot{n}_e$. This would imply that tidal friction has been nearly constant and that it is the non-frictional effects that have done most of the changing. However, this conclusion is far from being established, and there is no known theoretical reason that requires it.

Summary

Our knowledge of the accelerations of the sun, moon, and earth has changed drastically within the past five years. The values of $\dot{n}_M$ and $\dot{n}_e$ that most people accepted five years ago were little more than half of those given in Eqs. (13). Further, it

[h] The values in Eqs. (11), (12), and (13) are also mean rather than current values.

was commonly asserted that the mechanism of tidal friction had been established, although some authorities strongly demurred from this position. Some writers had even "proved" that the accelerations have been nearly constant within the historical period.

We know now that existing theories of tidal friction are seriously inadequate, and we know that at least some non-gravitational effects have changed by large amounts within less than 1000 years of the present. We do not know whether the changes have been in frictional effects, in non-frictional effects, or in both.

Since we are dealing with ancient astronomy, it may seem paradoxical to say that our increased knowledge is a result of an increased volume of data, but this is the case. Actually, the change in the past few years has not been in the data that are known, but in the data that someone has taken the trouble to analyze. The volume of data that has been analyzed has increased by perhaps an order of magnitude in the past 5 years. In the history of the physical sciences, such an explosive growth in the volume of data has often been accompanied by a growth in understanding. We may expect this growth to continue in the immediate future.

Thus we stand at an exciting point in the development of this subject. Within the next decade, we may reasonably hope to have a sound understanding of the non-gravitational forces within the sun-earth-moon system, the forces that probably govern the evolution of the system within astronomical time.

TABLE 1

NOTATION

When using the following list for a symbol with a subscript, look up the symbol and the subscript separately. Two letters are used both as main symbols and as subscripts. Letters of the Latin alphabet are given first, in alphabetical order, following by letters of the Greek alphabet in alphabetical order. A prime following a symbol denotes differentiation with respect to τ. A dot over a symbol denotes differentiation with respect to t.

- D—in Eqs. (10) only, denotes a time interval measured in days. Elsewhere, it denotes the lunar elongation, which is the angular separation of the sun and moon.
- e —as a subscript, identifies the earth. Otherwise, it is used only as an auxiliary variable in Eqs. (10).
- I —a moment of inertia.
- M—as a subscript, identifies the moon. Otherwise, it is used only for the mean anomaly in Eqs. (10).
- n —equals $d\theta/dt = \dot{\theta}$.
- S —a subscript to identify the sun.
- s —a generalized time base.
- T —a torque.
- t —the dynamical time base defined by the equations of motion of the solar system.
- θ —an angular coordinate.
- λ —celestial longitude.
- ν —equals $d\theta/d\tau = \theta'$.
- τ —solar time; the time of ordinary civil life.

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[1] Published annually by the U.S. Government Printing Office, Washington, D.C., 20402.

[2] See R. B. Kershner, "Present State of Navigation by Doppler from Near Earth Satellites," *APL Technical Digest* **5**, No. 2, Nov.–Dec. 1965, 2–9.

[3] R. R. Newton, "An Observation of the Satellite Perturbation Produced by the Solar Tide," *Journal of Geophysical Research,* **70**, 1965, 5983–5989, and "A Satellite Determination of Tidal Parameters and Earth Deceleration," *Geophysical Journal of the Royal Astronomical Society,* **14**, 1968, 505–539.

[4] I am greatly over-simplifying the subject here. The interested reader can find more information in *The Earth* by Sir Harold Jeffreys, Cambridge University Press, Cambridge, 5th Edition, 1970, or in *The Rotation of the Earth,* by W. H. Munk and G. J. F. MacDonald, Cambridge University Press, Cambridge, 1960.

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[6] T. C. van Flandern, "The Secular Acceleration of the Moon," *Astronomical Journal,* **75**, 1970, 657–658; C. Oesterwinter and C. J. Cohen, "New Orbital Elements for Moon and Planets," *Celestial Mechanics,* **5**, 1972, 317–395; L. V. Morrison, "The Rotation of the Earth from AD 1663–1972 and the Constancy of G," *Nature,* **241**, 1973, 519–520.

[7] R. R. Newton, *Ancient Astronomical Observations and the Accelerations of the Earth and Moon,* Johns Hopkins Press, Baltimore, 1970.

[8] R. R. Newton, "The Earth's Acceleration as Deduced from al-Biruni's Solar Data," *Memoirs of the Royal Astronomical Society,* **76**, 1972, 99–128.

[9] R. R. Newton, *The Validity of Ephemeris Time Between –567 and 1018,* in preparation, intended for submission to the Johns Hopkins Press.

[10] Quoted by C. Ptolemy in Chapter VII.2 of *'E Mathematike Syntaxis,* commonly called *Almagest,* written about the year 142. The standard edition of the text is by J. L. Heiberg in *C. Ptolemaei Opera Quae Exstant Omnia,* B. G. Teubner, Leipzig, 1898. K. Manitius has translated this text into German (B. G. Teubner, Leipzig, 1913). The reader should note that there are several errors in Manitius's statements of dates. In the summary of Timocharis's observation, I have changed the dates and some other items into modern terminology.

[11] R. R. Newton, *Medieval Chronicles and the Rotation of the Earth,* Johns Hopkins Press, Baltimore, 1972.

[12] C. F. Martin, "A Study of the Rate of Rotation of the Earth from Occultations of Stars by the Moon, 1627–1860," a dissertation presented to Yale University, 1969; expected to be published in the *Astronomical Papers* series of the Naval Observatory.