Geo-Location Using Direction Finding Angles

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Passive geo-location of ground targets is commonly performed by surveillance aircraft using direction finding angles. These angles define the line of sight from the aircraft to the target and are computed using the response of an antenna array on the aircraft to the target's RF emissions. Direction finding angles are the inputs required by a geo-location algorithm, which is typically an extended Kalman filter or a batch processor. This modality allows a single aircraft to detect, classify, and localize ground-based signal sources. In this article, the direction finding angles used for geo-location are defined and a mathematical model is developed that relates measurements of these angles to the target's position on Earth. Special emphasis is given to the angle measurement provided by a linear antenna array. An algorithm is presented that uses iterated least squares to estimate a target's position from multiple angle measurements. Simulation results are shown for a single aircraft locating a stationary ground target.

INTRODUCTION

The detection, classification, and geo-location of ground-based signal sources is one of the primary missions of modern military surveillance aircraft. Example signals of interest are the search radar of an enemy missile site, the radio communications between enemy combatants, and the distress radio beacon being transmitted by a downed friendly pilot. The ability to locate such signal sources is of obvious importance, especially during times of conflict.

The geo-location of ground targets is performed by surveillance aircraft using the output of sensors attached to the aircraft. In order to determine the target's location on Earth, the sensor measurements are processed by some form of estimation algorithm. This geo-location algorithm provides both a location estimate and an estimation error covariance matrix. The covariance matrix gives a statistical representation of the uncertainty in the location estimate and is used to construct a target location confidence region, such as an "error ellipse," which can be displayed to the signals intelligence system operator on the aircraft or in a remote ground station. Geo-location performance is driven by the number of measurements, measurement error statistics, and engagement geometry.

Active sensors using radar and imaging techniques provide accurate measurements of range, range rate, position, and velocity of ground targets. However, these sensors do not make direct use of the target's transmitted signal and therefore do not distinguish between (say) a radar and a radio. To determine the location of a specific radar, the output from active sensors must be correlated with measurements from passive sensors that process and identify the signal type. Also, by transmitting energy in order to produce the measurements needed for geo-location, an active system can reveal the presence of the host platform to hostile forces.

Passive systems can detect and locate a signal source without the assistance of an active system and without revealing the presence of the host platform. When compared with active systems, passive systems typically provide lower geo-location accuracy but are less complicated and lower in weight. A passive system requires only one or more antennas, a receiver, and the necessary signal-processing hardware and software. Such systems are used on platforms ranging from small unmanned aerial vehicles (UAVs) to large surveillance aircraft.

One method of passive geo-location involves the target's signal being received by at least two aircraft. The collected data are then transmitted to a common processing node where measurements of time difference of arrival (TDOA) and frequency difference of arrival (FDOA)[1] are extracted for each pair of aircraft. When the engagement geometry is favorable, the isograms associated with the TDOA and FDOA measurements will have a near-orthogonal intersection, which allows a target location estimate to be computed with only one pair of measurements. High geo-location accuracy can be achieved using TDOA and FDOA measurements, but doing so requires the coordinated operation and time synchronization of multiple collection platforms within the area of operation.

Another sensor measurement that can be generated passively and used for geo-location is a direction finding (DF) angle. A DF angle is one of several possible angles used to define the line of sight (LOS) from the aircraft to the signal source and is computed using the response of an antenna array, phase interferometry, and signal processing. In order to be completely specified, the LOS must be resolved into two angles such as azimuth and elevation relative to a coordinate frame attached to the array. Depending upon the array configuration, the DF system may or may not be able to determine both angles. A planar array is able to measure both angles, but a linear array measures only the conical angle between the axis of the array and the LOS, as discussed in the next section. In this article, we refer to azimuth, elevation, and the conical angle as DF angles.

One advantage of using DF angles for geo-location is that the signal processing and geo-location algorithms involved are less complicated than those needed to process TDOA and FDOA measurements. The primary advantage, however, is that the use of DF angles allows a single aircraft, operating independently, to detect a signal, identify its type, and locate its source on the ground. The use of DF angles for geo-location is therefore very common and critically important.

The purpose of this article is to present the mathematical model of DF angles needed for geo-location algorithm development. Because of their widespread use, we first discuss linear antenna arrays and their associated DF angle measurement. Following this, the geometrical relationship between DF angles and position on Earth is developed. We then derive the measurement matrix needed by a geo-location algorithm. Finally, simulation results are shown for a single surveillance aircraft attempting to locate a stationary ground target.

LINEAR ANTENNA ARRAYS AND DF ANGLES

Consider a linear antenna array mounted along the longitudinal axis of an aircraft and a signal source located such that the angle between this axis and the LOS is λ. Neglecting interactions between the signal and the aircraft body, the phase response across the array for this signal source will also be produced by any emitter located such that the conical LOS angle is λ.[2] Processing the phase response produces a cone of ambiguous directions from the aircraft to the possible target locations, as illustrated in Figs. 1 and 2.

In these figures, the coordinate frame shown is a forward-right-down frame attached to the aircraft, the target is the red dot, and the LOS is the red line. The target is located at an azimuth of 20° and an elevation of –30°, which gives a conical LOS angle λ of approximately 35.5°. This is the angle between the *x* axis and the red line and is also the angle between the *x* axis and any point on the cone.

Any emitter located such that its LOS is along the cone will produce the same phase response across the array as the emitter located at this specific azimuth and

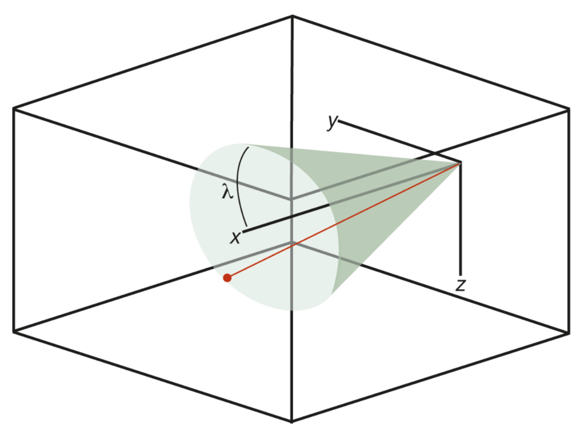

Figure 1. 3-D view of cone of possible emitter locations.

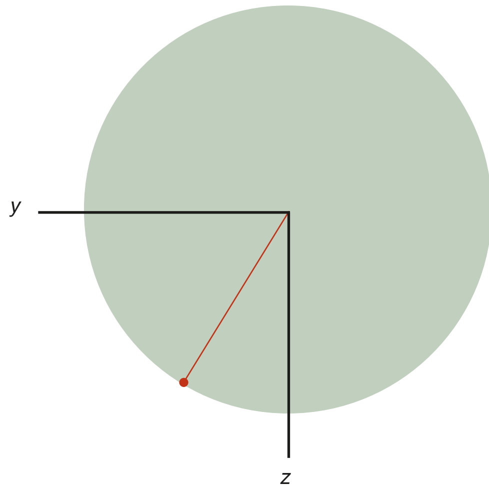

Figure 2. Front view of cone of possible emitter locations.

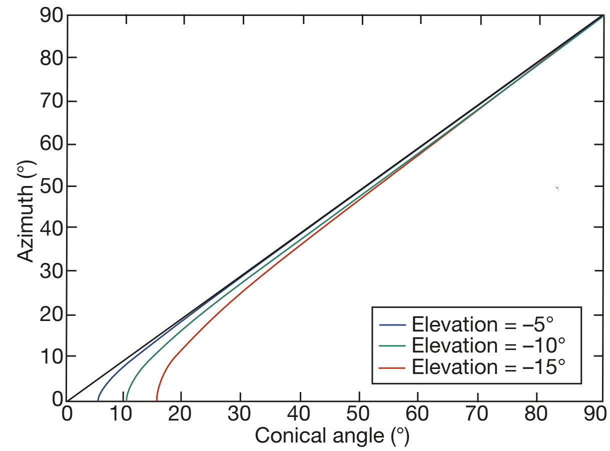

Figure 3. Azimuth versus conical angle for the entire FOV.

elevation. As a result, it is not possible in this situation to determine the target's azimuth and elevation when using a linear antenna array and only the array phase response. However, when the aircraft operates at a significant distance from the signal source, the elevation angle to the target will be small in magnitude. If during such a "standoff" mission the azimuth angle is close to 90°, then azimuth can be accurately approximated by the measured conical LOS angle and used solely for geo-location. The lack of elevation measurements is not detrimental—they would provide little additional information because they are known to be close to zero in magnitude.

The relationship between azimuth angle and the conical LOS angle is shown in Figs. 3 and 4 for elevation angles of –5°, –10°, and –15°. Figure 3 shows this relationship for an azimuth angle between 0° and 90° and is labeled as representing the entire field of view (FOV) of the array. The entire FOV actually consists of azimuth angles from 0° to 180°, but the plot is symmetrical about 90°. The range of azimuth angles close to 90° is often referred to as the "primary FOV." Figure 4 shows a primary FOV defined for azimuth angles between 60° and 90°.

The DF system on a surveillance aircraft using a linear array for geo-location is calibrated using flight and/or anechoic chamber data. These data consist of the measured array phase response for signal sources at known frequencies across a range of known azimuth and elevation angle values. The collected phase responses will include the effects of signal interaction with the aircraft body and will be unique to each aircraft. The magnitude of elevation angles used during data collection will typically be small, say, no greater than 15°, so that the conical LOS angle and azimuth will be close in value within the primary FOV. The data are then used to develop a mathematical model that gives the DF angle, either the conical angle or azimuth, and its variance as a function of phase response. This mathematical model and the calibration data become part of the system's software and are used operationally to compute the DF angle for each received signal, which includes the resolution of left–right and front–back ambiguities. The DF angle and its variance are the inputs required by a geo-location algorithm.

Restricting the elevation angle magnitude to 15° does not limit the operational capabilities of a surveillance aircraft as much as might be expected. Consider an aircraft at an altitude of 30,000 ft. The ground range from the aircraft to a location on Earth at an elevation angle of –15° is 18 nautical miles. At the other extreme, the elevation angle to the horizon is –3° and the associated ground range is 184 nautical miles. Depending upon signal frequency and atmospheric conditions, the actual "radio horizon" can be much greater than 184 nautical miles. Therefore, calibrating a linear array for elevation angles between –15° and –3° allows an aircraft in level flight at 30,000 ft to compute DF angles and locate targets at ground ranges between approximately 20 and 200 nautical miles. These ranges are reasonable for surveillance aircraft with a standoff mission. Aircraft operating closer to the target signal source require a lower altitude and/or a different antenna array configuration.

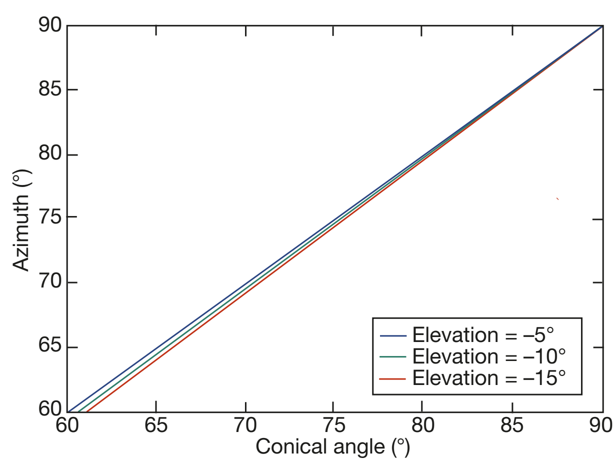

Figure 4. Azimuth versus conical angle for the primary FOV.

If a signal source is located at an elevation angle outside of the range used for calibration, then the received signal will produce a phase response across the array that is a poor fit to the calibration data. In this situation, either the signal will be rejected or it will be used but with a large variance assigned to the computed DF angle. If this DF angle is used for geo-location, the large variance assigned will cause it to be "de-weighted" when used with other measurements by the geo-location algorithm. Other geo-location methods, such as probabilistic data association,[3] perform probabilistic weighting of the measurements in order to account for the case where a non-target-originated measurement is accepted. These methods are of practical interest[4] but are not considered in this article.

Angle of arrival (AOA) is a term that, unfortunately, is applied to both azimuth and the conical LOS angle. The definition of AOA seems to vary with each system and aircraft, resulting in confusion. For the remainder of this article, we will define AOA to be the conical LOS angle λ to distinguish it from azimuth. Direction of arrival (DOA) is the angle equivalent to azimuth or AOA when defined relative to a local-level coordinate frame at the current aircraft position. DOA is computed using azimuth or AOA, an estimate of the elevation angle to the target, the antenna array mounting angles on the aircraft, and aircraft inertial navigation system (INS) output. A simplified relationship between AOA and DOA is illustrated in Fig. 5.

The geo-location systems on some surveillance aircraft depend entirely upon DF angle measurements produced by a linear antenna array, i.e., they do not have access to other measurement types such as range, range rate, TDOA, or FDOA. These systems will often convert the measured DF angles to DOA and use DOA for geo-location. One advantage of using DOA measurements is that it simplifies the geo-location algorithm: the estimation problem can be solved in a plane tangent to Earth's surface instead of in 3-D space relative to a spherical or ellipsoidal Earth model. Another advantage is that a planar algorithm using DOA for geo-location is more stable than a 3-D algorithm using azimuth when there are few measurements and the true DOA changes little during the collection. A planar algorithm may produce a solution in situations where a 3-D algorithm will diverge. A disadvantage is that additional coordinate transformations are required. A plane tangent to Earth's surface is constructed at some point within the area of operation, typically at the initial aircraft position. The aircraft positions and DOA angles are projected onto this plane, the geo-location problem is solved, and then the planar target location is projected onto the Earth model to produce a 3-D estimate.

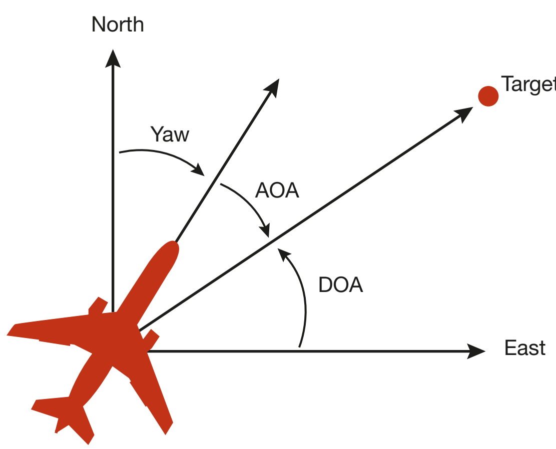

Figure 5. AOA and DOA.

If DF angle measurements are to be used with other measurement types, then there is no advantage in using DOA instead of azimuth or AOA and attempting to solve the geo-location problem in a local tangent plane. The reason for this is that it is difficult, if not impossible, to generate accurate planar approximations of measurement types such as TDOA and FDOA. The mathematical model needed for geo-location using DOA will not be developed in this article.

COORDINATE FRAMES

The following Cartesian coordinate frames are defined to support the processing of DF angle measurements: E, Earth-Centered-Earth-Fixed (ECEF); N, North-East-Down at the aircraft; B, aircraft body forward-right-down; and A, antenna array. In this article, a left superscript is used to indicate the coordinate frame in which a vector is represented. For example, ${}^{E}\boldsymbol{p}$ is the vector $\boldsymbol{p}$ when represented in the ECEF frame E.

The ECEF frame E has its z axis through the North Pole, its x axis through the intersection of the equator and the Greenwich Meridian, and its y axis oriented to create a right-handed coordinate system. The North-East-Down frame N has its x axis directed north along the local longitude line, its y axis directed east, and its z axis directed down along the local vertical. The aircraft body frame B has its x axis directed forward out of the nose of the aircraft, its y axis directed to the right from the pilot's perspective, and its z axis directed down out of the bottom of the aircraft. The antenna array frame A is oriented such that the LOS is along its x axis when AOA is zero.

The World Geodetic System 1984 (WGS84)[5] models the Earth's surface as an oblate spheroid (ellipsoid), which allows Cartesian ECEF positions on Earth's surface to be represented using the angles longitude and geodetic latitude. The WGS84 was developed by the National Imagery and Mapping Agency, now the National Geospatial-Intelligence Agency, and has been accepted as a standard for use in geodesy and navigation.

ROTATION MATRICES

The matrices associated with a rotation of (say) δ about the x, y, and z axes of a coordinate frame are

$$R(\delta, x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\delta) & -\sin(\delta) \\ 0 & \sin(\delta) & \cos(\delta) \end{bmatrix},$$

$$R(\delta, y) = \begin{bmatrix} \cos(\delta) & 0 & \sin(\delta) \\ 0 & 1 & 0 \\ -\sin(\delta) & 0 & \cos(\delta) \end{bmatrix},$$

$$R(\delta, z) = \begin{bmatrix} \cos(\delta) & -\sin(\delta) & 0 \\ \sin(\delta) & \cos(\delta) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The relative orientation of frames E and N is defined in terms of aircraft longitude ψ_{ac} and geodetic latitude θ_{ac} using the following rotation matrix:

$$TEN = R(\psi_{\text{ac}}, z)R\left(-\theta_{\text{ac}} - \frac{\pi}{2}, y\right).$$

The notation "*TEN*" is interpreted to mean the **T**ransformation to frame **E** from frame **N**, i.e., ${}^{E}\boldsymbol{p} = TEN \cdot {}^{N}\boldsymbol{p}$, for any vector $\boldsymbol{p}$. The relative orientation of frames N and B is defined using aircraft INS output

$$TNB = R(yaw, z)R(pitch, y)R(roll, x).$$

The relative orientation of frames B and A is defined by the antenna array mounting angles α, β, and γ using

$$TBA = R(\alpha, z)R(\beta, y)R(\gamma, x).$$

The relative orientation of any two coordinate frames can be found by multiplying the appropriate rotation matrices. For example, the relative orientation of frames E and B is given by $TEB = TEN \cdot TNB$. Also, the inverse of any rotation matrix is given by its transpose. For example, $TNE = [TEN]^T$.

MEASUREMENT MODEL

Let ψ and θ represent the unknown target WGS84 longitude and geodetic latitude, respectively. We will assume that target altitude a is either known or can be computed as a function of longitude and latitude using Digital Terrain Elevation Data. The target location in ECEF coordinates is given by the following 3 × 1 vector:[6]

$$\boldsymbol{p}_{\text{tgt}} = \begin{bmatrix} (r_{east} + a)\cos(\psi)\cos(\theta) \\ (r_{east} + a)\sin(\psi)\cos(\theta) \\ \left(r_{east}(1 - \varepsilon^2) + a\right)\sin(\theta) \end{bmatrix},$$

where

$$r_{east} = \frac{r_{\text{eq}}}{\sqrt{1 - \varepsilon^2 \sin^2(\theta)}}$$

is Earth's transverse radius of curvature, ε is Earth's eccentricity, and r_{eq} is Earth's equatorial radius. Note that the left superscript E has been suppressed to simplify notation. The position of the target relative to the aircraft in ECEF coordinates is

$${}^{E}\boldsymbol{p} = \boldsymbol{p}_{\text{tgt}} - \boldsymbol{p}_{\text{ac}},$$

where $\boldsymbol{p}_{\text{ac}}$ is the aircraft's position in ECEF coordinates. This relative position vector in frame A is

$${}^{A}\boldsymbol{p} = TAE \cdot {}^{E}\boldsymbol{p}.$$

A unit vector along the LOS in frame A is

$${}^{A}\boldsymbol{u} = \frac{1}{\|{}^{A}\boldsymbol{p}\|}{}^{A}\boldsymbol{p},$$

where $\|{}^{A}\boldsymbol{p}\|$ is the range from the aircraft to the target. Let φ, η, and λ represent the DF angles azimuth, elevation, and AOA, respectively. Then

$${}^{A}\boldsymbol{u} = R(\varphi, z)R(\eta, y)\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\varphi)\cos(\eta) \\ \sin(\varphi)\cos(\eta) \\ -\sin(\eta) \end{bmatrix}.$$

This definition of azimuth and elevation is such that if frame A is aligned with frame B, then a positive azimuth indicates that the target is to the right of the pilot, and a negative elevation indicates that the target is below the pilot. The DF angles are related to the unit vector ${}^{A}\boldsymbol{u}$ by

$$\varphi = \tan^{-1}\left(\frac{{}^{A}\boldsymbol{u}(2)}{{}^{A}\boldsymbol{u}(1)}\right)$$

$$\eta = \tan^{-1}\left(\frac{-{}^{A}\boldsymbol{u}(3)}{\sqrt{{}^{A}\boldsymbol{u}(1)^2 + {}^{A}\boldsymbol{u}(2)^2}}\right)$$

$$\lambda = \tan^{-1}\left(\frac{\sqrt{{}^{A}\boldsymbol{u}(2)^2 + {}^{A}\boldsymbol{u}(3)^2}}{{}^{A}\boldsymbol{u}(1)}\right).$$

Now let f_i be the function defined by the calculations shown above such that $f_i(\boldsymbol{p}_{\text{tgt}})$ gives the true value of azimuth, elevation, or AOA at aircraft position i for $i = 1, 2, \ldots n$, where n is the number of DF angle measurements. Each DF angle is also a function of the aircraft position vector $\boldsymbol{p}_{\text{ac}}$, but this dependency has been suppressed to simplify notation.

The 2 × 1 vector to be estimated is

$$\boldsymbol{x} = \begin{bmatrix} \psi \\ \theta \end{bmatrix}.$$

Define the measurement function h_i such that

$$h_i(\boldsymbol{x}) = f_i(\boldsymbol{p}_{\text{tgt}}(\boldsymbol{x})),$$

and let z_i be the DF angle measurement. Then

$$z_i = h_i(\boldsymbol{x}) + v_i,$$

where v_i is the measurement error. If we assume that these errors are unbiased, uncorrelated, and Gaussian with known variances, then

$$v_i \sim \mathcal{N}(0, \sigma_i^2),$$

where σ_i is assumed to be known for each i. This error model is obviously idealistic, but it is sufficient for geo-location when the DF system has been properly calibrated. The measurement model in vector form is

$$\boldsymbol{z} = \mathbf{h}(\mathbf{x}) + \mathbf{v}$$

$$\boldsymbol{v} \sim \mathcal{N}(0, \mathbf{R}),$$

where

$$\boldsymbol{R} = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_n^2 \end{bmatrix}$$

is the $n \times n$ positive definite measurement error covariance matrix.

GEO-LOCATION

Determining the target's location involves using the given measurement model to compute an estimate $\hat{\boldsymbol{x}}$ of the parameter vector $\boldsymbol{x}$ defined in *Measurement Model*. Doing so is equivalent to solving a nonlinear optimal estimation problem. The algorithms used to solve such problems fall into one of two broad categories: recursive or batch. A recursive algorithm processes only the most recent measurement to refine the estimate of the unknown parameter vector computed from the previous measurement. A batch algorithm processes all measurements simultaneously to produce one estimate. There are advantages and disadvantages of each approach. In general, batch algorithms are more stable than recursive algorithms but require more data storage and computation time. The recursive algorithm typically used for geo-location is the extended Kalman filter.[7–9] Examples of batch processors are iterated least-squares (ILS) and the Levenberg–Marquardt algorithm.[10, 11]

Both recursive and batch geo-location algorithms require an initial estimate $\hat{\boldsymbol{x}}_0$ that can be computed for DF angles as follows. Using two or more of the early azimuth or AOA measurements, compute the associated DOA values. Using these angles and Brown's closed-form algorithm,[1] compute a planar target location estimate. Project this estimate onto the WGS84 Earth model to form initial estimates of target longitude and latitude.

If we treat the parameter vector $\boldsymbol{x}$ as an unknown constant instead of a random variable, then the assumption that the measurement errors are Gaussian gives that the maximum-likelihood estimate[12] of $\boldsymbol{x}$ is

$$\hat{\boldsymbol{x}} = \arg\min_x (\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x}))^T \boldsymbol{R}^{-1} (\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})).$$

This is the parameter estimate for which the observed measurements are most likely to have been produced by the assumed model. This value can be found using batch processing and ILS, which consists of the iteration

$$\hat{\boldsymbol{x}}_{k+1} = \hat{\boldsymbol{x}}_k + \left[\boldsymbol{H}_k^T \boldsymbol{R}^{-1} \boldsymbol{H}_k\right]^{-1} \boldsymbol{H}_k^T \boldsymbol{R}^{-1} (\boldsymbol{z} - \boldsymbol{h}_k),$$

with the initial estimate $\hat{\boldsymbol{x}}_0$ computed as described above. In the above expression, the $n \times 1$ measurement function is

$$\boldsymbol{h}_k = \boldsymbol{h}(\hat{\boldsymbol{x}}_k),$$

and the $n \times 2$ measurement matrix is

$$\boldsymbol{H}_k = \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{x}}(\hat{\boldsymbol{x}}_k).$$

The measurement function $\boldsymbol{h}$ was developed in the previous section, and the measurement matrix $\boldsymbol{H}$ will be developed in *Measurement Matrix*. Note that in each iteration k, $\boldsymbol{h}_k$ and $\boldsymbol{H}_k$ are computed using only aircraft positions and the current target location estimate $\hat{\boldsymbol{x}}_k$, i.e., the DF angle measurements are not used. Note also that the iteration involved is not the same as with a Kalman filter. With ILS, the iteration is repeated until some desired convergence tolerance has been met, such as $\|\hat{\boldsymbol{x}}_{k+1} - \hat{\boldsymbol{x}}_k\|$ being sufficiently small. With a Kalman filter, the iteration simply consists of one step for each new measurement. Also, the vector $\boldsymbol{z}$ used by ILS contains all measurements, so all measurements are being used in each step. A Kalman filter uses only the most recent measurement in each step.

MEASUREMENT MATRIX

The measurement matrix $\boldsymbol{H}$ is

$$\boldsymbol{H} = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_n \end{bmatrix}, \text{ where } H_i = \frac{\partial h_i}{\partial \boldsymbol{x}}.$$

Using the chain rule, this derivative can be expressed as the following product of five factors:

$$H_i = \frac{\partial h_i}{\partial {}^A\boldsymbol{u}} \cdot \frac{\partial {}^A\boldsymbol{u}}{\partial {}^A\boldsymbol{p}} \cdot \frac{\partial {}^A\boldsymbol{p}}{\partial {}^E\boldsymbol{p}} \cdot \frac{\partial {}^E\boldsymbol{p}}{\partial \boldsymbol{p}_{\text{tgt}}} \cdot \frac{\partial \boldsymbol{p}_{\text{tgt}}}{\partial \boldsymbol{x}}.$$

The first factor in this product is unique for each DF angle. For an azimuth measurement,

$$h_i = \varphi$$

$$\frac{\partial h_i}{\partial {}^A\boldsymbol{u}} = \frac{1}{\cos(\eta)}[-\sin(\varphi) \;\; \cos(\varphi) \;\; 0].$$

For an elevation measurement,

$$h_i = \eta$$

$$\frac{\partial h_i}{\partial {}^{A}\boldsymbol{u}} = [-\cos(\varphi)\sin(\eta) \quad -\sin(\varphi)\sin(\eta) \quad -\cos(\eta)].$$

For an AOA measurement,

$$h_i = \lambda$$

$$\frac{\partial h_i}{\partial {}^{A}\boldsymbol{u}} = \frac{1}{\sin(\lambda)}[-\sin^2(\lambda) \quad \sin(\varphi)\cos(\eta)\cos(\lambda) \quad -\sin(\eta)\cos(\lambda)].$$

The remaining factors are the same for each measurement type. Factors 2, 3, and 4 are

$$\frac{\partial {}^{A}\boldsymbol{u}}{\partial {}^{A}\boldsymbol{p}} = \frac{1}{\| {}^{A}\boldsymbol{p} \|}\left[\boldsymbol{I}_{3\times 3} - {}^{A}\boldsymbol{u}\,{}^{A}\boldsymbol{u}^{T}\right],$$

$$\frac{\partial {}^{A}\boldsymbol{p}}{\partial {}^{E}\boldsymbol{p}} = TAE, \text{ and}$$

$$\frac{\partial {}^{E}\boldsymbol{p}}{\partial \boldsymbol{p}_{\text{tgt}}} = \boldsymbol{I}_{3\times 3}.$$

Factor 5 is $\frac{\partial \boldsymbol{p}_{\text{tgt}}}{\partial \boldsymbol{x}} = \left[\frac{\partial \boldsymbol{p}_{\text{tgt}}}{\partial \psi} \quad \frac{\partial \boldsymbol{p}_{\text{tgt}}}{\partial \theta}\right]$, where

$$\frac{\partial \boldsymbol{p}_{\text{tgt}}}{\partial \psi} = \begin{bmatrix} \left(\frac{\partial a}{\partial \psi}\right)\cos(\psi)\cos(\theta) - (r_{east} + a)\sin(\psi)\cos(\theta) \\ \left(\frac{\partial a}{\partial \psi}\right)\sin(\psi)\cos(\theta) + (r_{east} + a)\cos(\psi)\cos(\theta) \\ \left(\frac{\partial a}{\partial \psi}\right)\sin(\theta) \end{bmatrix},$$

$$\frac{\partial \boldsymbol{p}_{\text{tgt}}}{\partial \theta} = \begin{bmatrix} \left(\frac{\partial r_{east}}{\partial \theta} + \frac{\partial a}{\partial \theta}\right)\cos(\psi)\cos(\theta) - (r_{east} + a)\cos(\psi)\sin(\theta) \\ \left(\frac{\partial r_{east}}{\partial \theta} + \frac{\partial a}{\partial \theta}\right)\sin(\psi)\cos(\theta) - (r_{east} + a)\sin(\psi)\sin(\theta) \\ \left(\frac{\partial r_{east}}{\partial \theta}(1-\varepsilon^2) + \frac{\partial a}{\partial \theta}\right)\sin(\theta) + \left(r_{east}(1-\varepsilon^2) + a\right)\cos(\theta) \end{bmatrix}, \quad \text{and}$$

$$\frac{\partial r_{east}}{\partial \theta} = \frac{\varepsilon^2 \sin(\theta)\cos(\theta) r_{east}}{1 - \varepsilon^2 \sin^2(\theta)}.$$

If the target altitude a is constant, then

$$\frac{\partial a}{\partial \psi} = \frac{\partial a}{\partial \theta} = 0.$$

Otherwise, these values are computed numerically as gradients of the terrain data.

ESTIMATION ERROR COVARIANCE MATRIX

Let $\hat{\boldsymbol{x}}$ be the target location estimate after convergence of ILS, and let

$$\boldsymbol{H} = \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{x}}(\hat{\boldsymbol{x}}).$$

The covariance matrix for the target's location error in WGS84 coordinates (ψ, θ) is given by

$$P_x = E[(x-\hat{x})(x-\hat{x})^T] = [H^T R^{-1} H]^{-1},$$

where $E[\cdot]$ is the statistical expectation operator. A linear approximation of the relationship between the target's location in ECEF coordinates (x, y, z) and WGS84 coordinates is given by the truncated Taylor series expansion

$$(p_{\text{tgt}} - \hat{p}_{\text{tgt}}) = \left[\frac{\partial p_{\text{tgt}}}{\partial x}\right](x-\hat{x}).$$

Using the above approximation, the covariance matrix for the target's location error in ECEF coordinates is

$$P_{p_{\text{tgt}}} = E\left[(p_{\text{tgt}}-\hat{p}_{\text{tgt}})(p_{\text{tgt}}-\hat{p}_{\text{tgt}})^T\right] = \left[\frac{\partial p_{\text{tgt}}}{\partial x}\right] P_x \left[\frac{\partial p_{\text{tgt}}}{\partial x}\right]^T.$$

To construct an error ellipse, the above ECEF covariance matrix is rotated into the coordinate frame used by the signals intelligence operator's workstation, the appropriate 2 × 2 submatrix is extracted, and an eigen decomposition is performed to determine the length and direction of the ellipse axes.

PARAMETER VALUES

In the above development, the quantities that are specific to each operational system are the antenna array mounting angles (α, β, γ) and the DF angle measurement error σ. These values will vary widely depending on the antenna array size and configuration, signal type, frequency, etc.

One common configuration is a single antenna array mounted along the fuselage of the aircraft that receives signals from both the left and right sides. In this case, reasonable values for the antenna array mounting angles are (α, β, γ) = (0, 0, 0). Another configuration is two arrays mounted on the aircraft such that one receives signals emitted from targets on the left side of the aircraft and the other receives from targets on the right. In this case, reasonable values for the array mounting angles are (α, β, γ) = (–90°, 0, 0) for the left array and (α, β, γ) = (90°, 0, 0) for the right array.

DF measurement errors are produced by a combination of factors such as antenna array configuration, signal frequency, noise levels, interactions between the signal and aircraft body, calibration data errors, and error in the approximation of the conical antenna response by a planar angle. For large linear arrays and measurements made within the primary FOV, an azimuth measurement error sigma in the range of 0.1° ≤ σ ≤ 1.0° is reasonable for a wide range of signal types and frequencies. Outside of the primary FOV, where AOA is a poor approximation of azimuth, the sigma values can be an order of magnitude higher.

Table 1. Simulation parameter values

Parameter	Units	Value
Initial range from aircraft to target	Nautical miles	100
Aircraft altitude	Feet	30,000
Aircraft ground speed	Knots	400
Mounting angle α	Degrees	90
Mounting angle β	Degrees	0
Mounting angle γ	Degrees	0
Number of azimuth measurements	—	10
Time between measurements	Seconds	90
Measurement error sigma	Degrees	0.1

SIMULATION EXAMPLE

The mathematical model developed in this article was simulated in MATLAB for a single surveillance aircraft attempting to locate a stationary ground target during a standoff mission. The parameter values used in the simulation are given in Table 1.

The true azimuth values are between –30° and 30°, which is a reasonable primary FOV for a linear array directed toward the right side of the aircraft. The azimuth

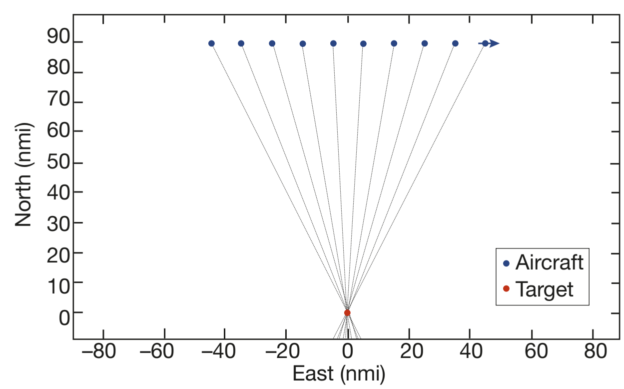

Figure 6. Surveillance geometry and lines of bearing.

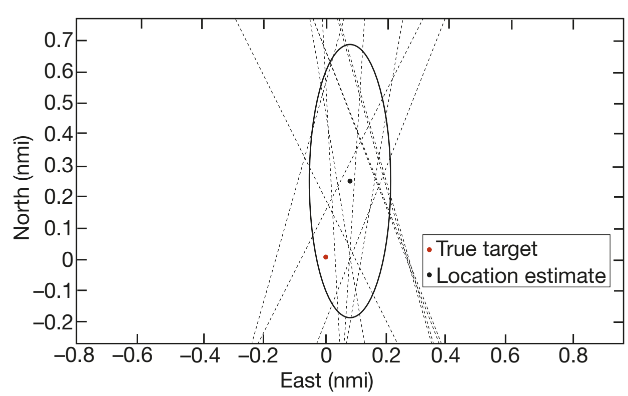

Figure 7. Final location estimate and 95% error ellipse.

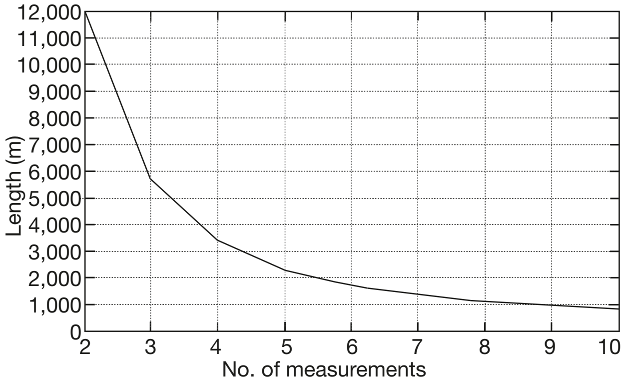

Figure 8. 95% elliptical error probable.

measurements were generated by adding to each true azimuth value a Gaussian random error having a mean of 0 and a sigma of 0.1°. The isogram associated with each azimuth measurement is a "line of bearing." The surveillance geometry and these lines are shown in Fig. 6. Note that the simulation was performed in WGS84 coordinates, as developed in this article, but the results are displayed in a local tangent plane for the sake of visualization.

The geo-location algorithm used in the simulation was ILS. The initial target location estimate was computed using the first two azimuth measurements and Brown's algorithm,[1] as described in the *Geo-Location* section. The final results are shown in Fig. 7.

A metric commonly used to quantify geo-location performance is elliptical error probable, which is the length of the semimajor axis of the error ellipse. This quantity is plotted in Fig. 8 to illustrate the improvement in geo-location performance as a function of number of measurements. The final value is approximately 820 m at the end of a 13.5-min flight.

CONCLUSION

The focus of this article was to present the mathematical model of DF angles needed for geo-location algorithm development. We defined DF angles and discussed the advantages of their use for geo-location when compared with other measurement types, both active and passive. The primary advantage of using DF angles is that it allows a single aircraft to passively detect, identify, and locate a ground signal source. A simulation example was used to quantify the geo-location performance that can be expected when an aircraft is using DF angles for geo-location during a standoff surveillance mission.

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