A Multistatic Performance Prediction Methodology

Julius I. Bowen and Ronald W. Mitnick

The Shallow Water Acoustic Technology Program of the Defense Advanced Research Projects Agency included development of simple impulsive acoustic sources for low-frequency submarine detection, which, in turn, raised questions about their efficient operational use. The multistatic performance prediction methodology was a response to that interest. It can be used to evaluate detection as a function of source and receiver densities. We discuss the underlying concepts and implementation of this methodology and give examples of its application. (Keywords: Active sonar, Distributed fields, Multistatics.)

BACKGROUND

The breakup of the Soviet Union and the Warsaw Pact in the late 1980s precipitated a major change in emphasis on the part of military planners in the United States. It was recognized at that time that the lack of a bipolar world order would result in the emergence of regional conflicts based on religion or ethnicity, conflicts that heretofore may have been prevented by superpower intervention motivated by the drive to gain global influence. For the U.S. Navy, these circumstances have resulted in a shift away from open-ocean operations against a strategic threat, the Soviet Union, to supporting expeditionary warfare being conducted in littoral environments.

Among the many difficult threats our forces face in the maritime littoral environment is the modern diesel-electric submarine. Submarines operating in a near-shore environment represent a stealthy lethal threat. Even without sophisticated quieting technology, a modern diesel-electric submarine operating in littoral waters is difficult to detect with any currently available sensing scheme because of the noise, clutter, and variability in the environment. The incorporation of advanced quieting technologies further increases the threat from the modern diesel-electric submarine, which can potentially launch several different types of weapons, from mines to anti-air missiles to cruise missiles, as well as the submarine's traditional weapon, the torpedo. In particular, today's torpedoes, which are produced in and marketed by more that 10 countries, have great destructive power. Submarine-launched heavyweight torpedoes can break some warships in half, and the proliferation of wake-homing torpedoes presents the potential of putting "fire-and-forget" weapons, which are highly lethal and difficult to counter, on adversary submarines.

In response to this threat, in 1991, the Defense Advanced Research Projects Agency (DARPA) initiated the Shallow Water Anti-Submarine Warfare Program. Two major goals of the program were the development of high-power, low-frequency impulsive sources for active sonar and the application of advanced computer reasoning technologies to perform single-ping classification on potential returns from these non–Doppler-sensitive sources. The motivation for selecting low frequency was the existence in shallow water of an optimum band for both transmission loss and bottom scattering in the frequency band nominally between 200 and 600 Hz. DARPA's impulsive source efforts complemented the Navy's Low Frequency Active Program, which was oriented toward the use of controlled waveform sources. The emphasis on single-ping classification reflected an operational consideration stemming from the requirement for a period of several minutes between pings, necessitated by reverberation characteristics of the high-power DARPA source. A series of sea tests was conducted to characterize the performance potential of the DARPA system concept, eventually leading to a transition of sponsorship from DARPA to the Navy.

APL's Submarine Technology Department has been a participant in DARPA's Shallow Water Acoustic Technology Program. The DARPA Program Manager, William Carey, was interested in the development of a low-frequency, impulsive, shallow-water, active sonar performance scoping model with sufficient rigor to reflect many of the geometric effects unique to this system concept. The remainder of this article describes the scoping model and details its development.

INTRODUCTION

The methodology to be discussed, multistatic performance prediction methodology (MPPM), evaluates the performance of submarine detection systems with dispersed acoustic sources and receivers, i.e., multistatic systems. MPPM is an alternative to several excellent Monte Carlo simulations that are available for assessing performance and examining tactics. Among the simulations currently used to analyze multistatic performance are the Multistatic Acoustic Simulation Model (MSASM), used by the Naval Air Weapon Center; the Sonar Equation Modeling and Simulation Tool (SEMAST), developed and used at the Naval Space and Warfare Systems Command Center; and the Surveillance Operational Concepts Model (SOCM), developed by Daniel H. Wagner Associates for APL, where it is extensively used. In common with these simulations, MPPM is based on evaluating the sonar equation; the acoustic environmental characterization may be likewise detailed. MPPM is based on probabilistic assumptions and constructs and, lacking explicit treatment of target actions, is not suited to evaluate tactics. Its utility comes from providing estimates of source and receiver densities needed to attain prescribed probabilities of submarine detection over time.

The methodology has evolved over the past few years, beginning with development of a version in response to a simplified problem posed by Carey. That problem and the responsive version of the methodology are discussed in the following section; we then present a treatment that is more generally applicable. Illustrative examples of applications are also given.

RESTRICTED PROBLEM/SIMPLIFIED SOLUTION

Imagine a large acoustically homogeneous area with a field of uniformly dispersed receivers and a patrolling submarine. How should acoustic sources be distributed over this region to detect the target? That is, how many impulsive sources are needed and how often should pings occur? Assume, for simplification, that the detection process is ambient noise limited (reverberation plays no role) and also that detection can occur only when the target heading provides "favorable" aspects to both the pinging source and to at least one of the receivers so that the target strength exceeds some threshold value. This occurs when target heading is such that the angle of "incidence" (the angle between the normal-to-the-target heading and the source) is near the angle of "reflection" to the receiver.

The target echo signal-to-noise ratio (SNR) from each ping at the beamformer output to each receiver is, following well-accepted convention,

$$\mathrm{SNR} = \mathrm{ESL} - \mathrm{TL}_{st} + \mathrm{TS} - \mathrm{TL}_{tr} - \mathrm{BN},$$

where

ESL = the energy source level over the spectral band that is being processed at the receiver,
TL = the transmission loss over the paths from source (s) to target (t) and from target to receiver (r),
TS (θ_s, θ_r) = the target strength (ratio of acoustic intensity from the source that is reflected toward the receiver to that striking the target), in general a function of the target's size and shape, frequency, and aspect angles from the source and receiver, and
BN = the beam noise (the amount of ambient noise in the beam containing the target).

If for some ping the SNR of at least one receiver exceeds a threshold value (called the detection threshold, DT), it is assumed that this receiver "detects" the target, the echo level being sufficient to recognize the target's presence. This is often a gross analytical simplification. Recognition of the target in a cluttered environment is a complex process and often cannot be done without sequentially exceeding the threshold level (or sensing some measurable Doppler shift in the echo). The methodology has been extended to allow for such cases, but that is not discussed here. In fact, as noted, in some experiments, exceeding the threshold one time might suffice for reliable classification of the echo.

The difference between the SNR and the DT is defined as signal excess, SE. That is, SE = SNR − DT, and detection is equivalent to SE > 0 for at least one receiver for at least one of the pings. This identification of SE > 0 with "detection" and SE ≤ 0 with "no detection" is sometimes referred to as a "cookie-cutter." More complex characterizations of detection, which treat detection probability as a function of SE, can be accommodated in MPPM in straightforward fashion, although this is not discussed here.

Figure 1 illustrates the vicinity of a target where receivers, if present, would have SE > 0. This example uses the ambient noise values and an empirical TL function that derive from measurements made during the DARPA-sponsored Acoustic Characterization Test (ACT) III of September 1995 in the shallow water in the Straits of Korea. The source-to-target distance is 10 nmi. (The figure also assumes values for several other terms in the sonar equation that are representative of components of active systems either already operational or from the world of research and development. Also assumed in this example is a specific value of TS representative of the target strength value that a small diesel submarine might present if fairly near "specular" reflection, a set of incident and reflection angles that might be found randomly in perhaps 20% of the cases.) The region where a receiver, if present, would result in positive SE is a circle with a "Pac-man"–like region excluded; because the example given is completely isotropic, the boundary of the region is defined by TL_{tr} = constant, which is the locus of points with SE = 0. Since the TL in this region is a monotonic function of range, the target-to-receiver distance is then a constant.

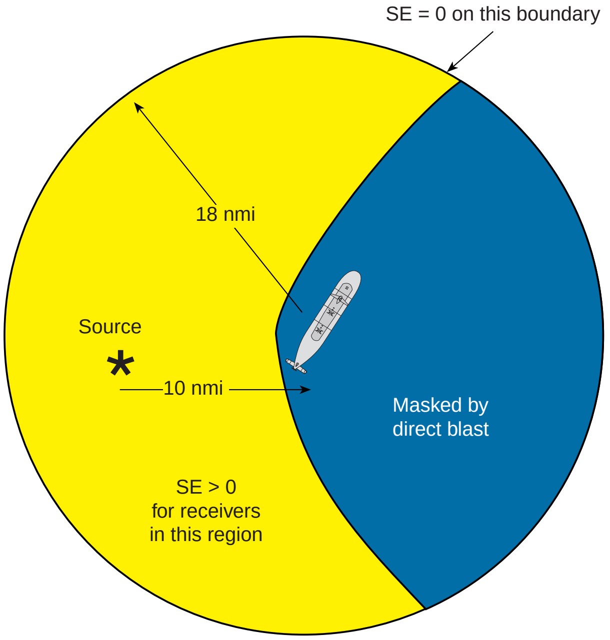

Figure 1. Example of a region with positive signal excess (SE) at the receiver, with noise-limited performance. Separation of source and target is 10 nmi; masking time is 5 s.

The excluded portion of the circle is caused by "direct blast masking." For a period of time after the source ping is initiated, acoustic energy with high intensity would make it impossible for a target echo to be detected if it were to arrive at the receiver during this time. Hence, for a fixed source-to-target distance and given "masking time interval," receivers in some portion of space cannot make detections even though the sonar equation would yield a positive SE. The time interval is at least as long as the ping itself, and is extended by energy delayed en route to the receiver over longer paths caused by multipath propagation and "out-of-plane" propagation.

The direct blast masking contour (and, hence, the fraction of the circle of Fig. 1 excluded from consideration for location of successful receivers) is determined by the equation

$$st + tr = sr + C\tau \ ,$$

where τ is the masking interval, C is the speed of sound, and the other terms denote distances from source to target (st), from target to receiver (tr), and from source to receiver (sr). Figure 2 shows a family of direct blast masking contours. These are functions of the source-to-target distance normalized by $C\tau$. The figure shows that the contours are close together when $st >> C\tau$ (the effect of masking is relatively small and changes slowly in this regime). When st becomes of order $C\tau$ and less, the masking grows relatively more significant; just when the SE > 0 contours might enclose large areas, most of those areas cannot be exploited as a result of masking. When $st \leq C\tau/2$, the target is completely masked by the direct blast, that is, the receiver is "deafened" by a direct blast much like an outfielder trying to catch a baseball while looking at the Sun.

Returning now to the original question: How should acoustic sources be distributed over this region to detect the target? In order to prevent the target from having a safe haven over the large patrol area, the sources will be uniformly spread over the entire search area (A) to do the best possible job of putting sound energy on the target. Assume a square grid of sources with spacing L. That is, the source density σ_s is $1/L^2$, or the "area per source," defined as the inverse of density, is L^2; hence, A/L^2 sources would be spread over an area A. (Area per source is used here simply to denote the inverse of source density. It does not imply that target detection is determined by only one of the sources dispersed in the search area.) If the target moves very slowly with respect to the "speed" at which adjacent source pings will occur ($v << L/t_0$, where t_0 is the time between adjacent pings and v is the target speed of advance), one may think of the target as (approximately) "frozen" in any "unit cell," a square L on edge containing both a source (at its center) and the target at some heading (uniformly distributed) and at some location, also uniformly distributed over this unit cell.

Now, for simplification, imagine that sources outside the unit cell containing the target are too distant to significantly contribute to detecting the target when compared to the source in the unit cell, the closest source. Consider the *average area* in which receivers (if present) would have positive SE *if* the target heading were such that TS exceeded the threshold TH; call this $A_R(L;\mathrm{TH})$, the effective receiver area. This area can be found by averaging over target headings and locations in the unit cell. To simplify the computation (we will remove this and some other simplifying assumptions in the next section), assume that we can approximate this area by setting TS *equal* to TH and, at the same time, multiply by the probability that TS exceeds TH. Thus,

$$A_R(L;\mathrm{TH}) = \int_0^{L/\sqrt{2}} a_R(l;\mathrm{TH})\,\omega(l)\,dl\,\mathrm{prob}(\mathrm{TS} > \mathrm{TH})\,, \quad (1)$$

where l is the distance from the target to the source in the center of the unit cell, and $a_R(l;\mathrm{TH})$ is the area with SE > 0 when TS = TH and source and target are separated by l. That distance, l, ranges over the unit cell, with probability density function $\omega(l)$, so that $\omega(l)\,dl$ is the probability that the source-to-target distance is between l and $(l + dl)$.

The receiver positions are totally unspecified, and the target heading is statistically independent of its location (i.e., the target has no "map" of source locations that allows it to use maneuvers to reduce TS to the next ping). The probability that TS > TH can be calculated by considering the probability that TS > TH for each source aspect angle, then averaging over source angles. That is, prob(TS > TH) is found directly from the function of source and receiver aspect angles TS (θ_s, θ_r).

This prescribes how $A_R(L;\mathrm{TH})$ is calculated. The effective receiver area is clearly a function of the system parameters and the environment (which determine the terms in the sonar equation), and the source density is implied in the integration limits. An example of A_R is given in Fig. 3, where it is plotted against the area per source (L^2) using the same TS threshold, TH, used in Fig. 1. Also plotted there is integration over position, i.e., before reducing by the probability that TS exceeds that threshold.

What is the relation of this quantity to detection performance? If the density of *receivers* is σ_R and the only source that would contribute to detection of a target is that within the unit cell, the expected number of receivers (n_R) with positive SE is $\sigma_R A_R$. This is the expected number of detections in a single ping cycle, i.e., pinging

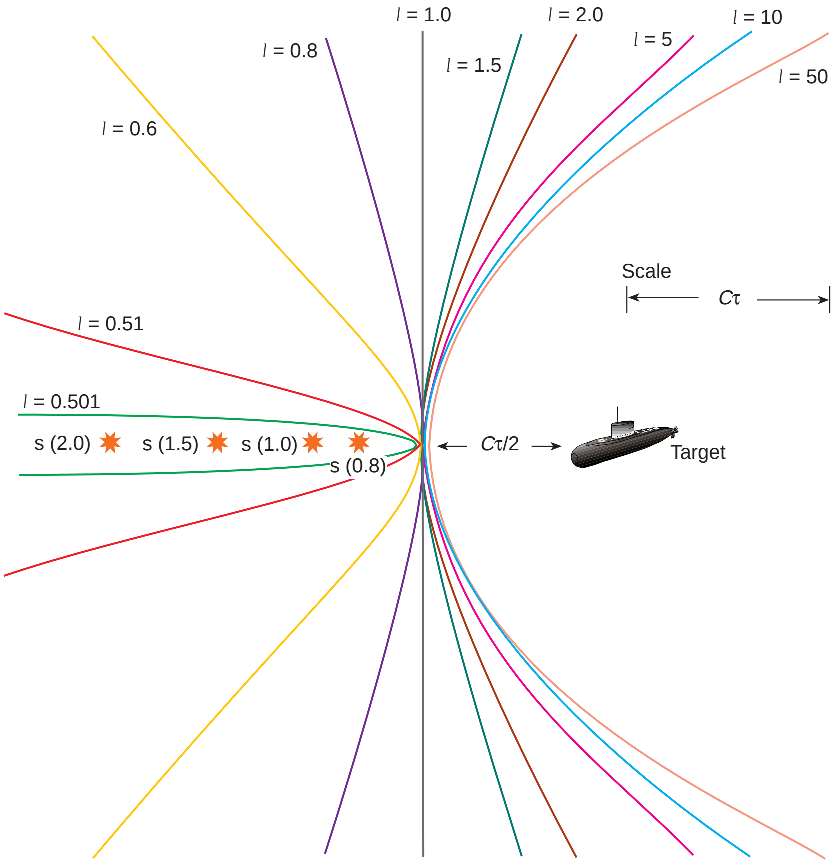

Figure 2. Direct blast masking contours. The source (s) is to the left of the target. The distances shown in parentheses are scaled with $C\tau$. The source-to-target distances, denoted by l, are similarly scaled. Receivers to the right of the contours plotted are masked by direct blast.

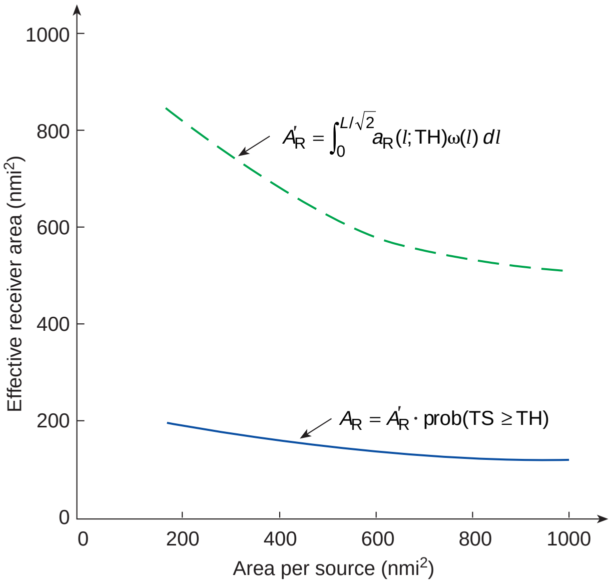

Figure 3. Example of effective receiver area A_R plotted against area per source (L^2) (single source, noise-limited performance).

around the field of sources once. That is, assuming that only the nearest source contributes to detecting the target (a not unreasonable assumption when TL is a monotonic function of *st* and the density of sources is not too great) ensures that one need only be concerned with the receivers "effective" from that ping in the unit cell containing the target.

Relating this single statistical measure, the expected number of detections, to the probability of detection (probability that at least one receiver gets contact, positive signal excess) requires an "invention." There is no "correct" distribution function of contacts because the effective receiver area is a theoretical construct (because of the averaging), not an identifiable locus of positions. Even if it were a physical area, there is no specified distribution of receivers, although reason would suggest that if the target is meandering over a large area A the receivers should be regularly spread out over the search area. To proceed, imagine first that there is only one receiver. Since the unit cell with the target is anywhere in A, one expects the probability of contact to be A_R/A and that of no contact in this ping cycle to be $(1 - A_R/A)$. Since we are interested in large search areas, $A_R/A << 1$, this is well approximated by $\exp(-A_R/A)$. If there were several receivers N spread over the search area, we would expect the probability of no contacts to be simply the product, $\exp(-NA_R/A)$. That is, the placement of a few receivers over an area that is large compared with A_R would suggest that they are (very nearly, at least) statistically independent. Since N/A is the density of receivers σ_R, then for a few receivers the probability of at least one detection becomes $1 - \exp(-\sigma_R A_R)$. We consequently *assume* that, to a good approximation, such a form will hold, and that the probability of success in one ping cycle, P_1, is given by

$$P_1 = 1 - e^{-\sigma_R A_R} \;, \tag{2}$$

as if the number of contacts had a Poisson distribution with expected value $n_R = \sigma_R A_R$. If there are m ping cycles and they are statistically independent attempts to detect the patrolling target, the probability becomes $1 - (1 - P_1)^m$ or

$$P_m = 1 - e^{-m\sigma_R A_R} \;. \tag{3}$$

Equation 3 represents a response to the question posed earlier under the many simplifying assumptions stated: The probability of detection (occurrence of at least one signal excess) in a "time" needed for m independent ping cycles is a simple function of the receiver density and A_R, a construct that depends on the source density and all the physics and system parameters. The independence of ping cycles requires at least some minimum time interval between cycles, e.g., about 1 h, but specifically depends on the dynamics of the patrolling target and the decorrelation times of various physical phenomena such as noise fluctuations. Note that when the ping cycles are independent of one another as has been assumed, there is simply an inverse relation between the number (or density) of receivers and the number of ping cycles.

As a concrete illustration of an application, with the same environment (Straits of Korea) used for Fig. 3 and the same TS threshold, suppose that a number of receivers are dispersed over 10,000 nmi^2. Then Eq. 3 permits one to estimate the number of sources (source pings per cycle) needed to attain a desired detection probability, say 0.9, in a prescribed number of ping cycles, for example m = 4 and m = 8. Results are given in Table 1.

Table 1. Number of sources for probability 0.9 in 10,000 nmi^2.

Number of receivers	4 ping cycles	8 ping cycles
20	(many!)	20
30	≈60	10
40	22	≈4
50	14	(few)
60	≈8	(few)

GENERALIZATION

Several assumptions identified in the foregoing section were made for the sake of simplifying computations and to illustrate the main ideas of MPPM. Some of these assumptions can be discarded; they are unduly restrictive, and fairly complex computations do not challenge contemporary personal computers. In particular, the unnecessary assumptions are as follows:

1. The performance is determined by ambient noise, i.e., reverberation plays no role.
   Comment: Reverberation is routinely predicted using various well-established codes; such predictions are readily incorporated in computing SE, hence in evaluating the areas with positive signal excess.
2. Contributions to the effective area come only from the source at the center of the unit cell containing the target in any ping cycle.
   Comment: All the sources spread about the search area may contribute to the area where receivers would have positive signal excess. For every source (whether or not it is in the unit cell containing the target), the area of positive signal excess for potential receivers may be computed for a specified target location in a unit cell and for a specified target heading, and the union of areas (the logical "or") can be computed once the source locations are specified. For a homogeneous environment, these locations will generally follow a regular grid, determined by the shape of the search area. As before, averaging over target positions in the unit cell and headings would follow. Multiple sources may imply a great many computations. Only a relatively small number of sources, however, usually contribute significantly to the effective receiver area. The other sources provide much less insonification of the target because they are too distant or are not at separations that lead to large amounts of acoustic energy reaching the target (convergence zone separations). Therefore, a bit of thought and precomputation will provide practical guidance to the number of sources to consider.
3. The calculation of SE, leading to evaluation of the effective area, is for a fixed TS.
   Comment: As implied in the previous paragraph, once the source configuration is given, every target heading determines the source and receiver aspect angles for each source ping and each potential receiver location; hence, TS is determined by the TS function. Values of TS may be much greater than a threshold value over some small range of equivalent aspect angle. The choice of threshold value for TS is somewhat arbitrary, and when TS is much greater than the threshold value, the effective receiver area contribution will be greater (perhaps much greater) than for the threshold value.

An example of the union of areas for three sources is shown in Fig. 4. The target heading in this figure is 45° (with respect to north), and the TS model corresponds to a medium-sized diesel-electric submarine. The reverberation scattering strength used in predicting reverberation is based on the aforementioned measurements, and the other environmental characterizations and system parameters are the same as for the previous example. The three sources are numbered to correspond to the numbers for their effective area contributions. The contributions to the effective receiver area in Fig. 4 appear discrete because the pixels used to represent the potential receiver locations have a 1-nmi resolution. Also, the extent of coverage is abridged as a result of program limitations. However, the dominant features are easy to understand. All the contributions are near "specular," i.e., the equivalent bistatic angle corresponds to near broadside to the target. The closer the source is to the target, the greater the contributions to effective area, i.e., TL increases monotonically with range in this environment, so nearby sources insonify the target more effectively. Note also that the union of areas for these three sources is the same as the sum of the areas. This is not true in general, but the assumption that the union can be replaced by the sum is frequently accurate and may make computations easier.

To capture the points above, suppose there are M sources. Then the effective receiver area can be written

$$A_{\mathrm{R}} = \Big\langle \Big\langle \bigcup_{i=1}^{\mathrm{M}} a_{\mathrm{R}i}(\vec{x};\varphi_{\mathrm{H}}) \Big\rangle_{\varphi_{\mathrm{H}}} \Big\rangle_{\vec{x}}\ , \qquad (4)$$

where $\bigcup_{i=1}^{\mathrm{M}}$ is the union of areas of positive signal excess for all of the M sources (or some dominant subset), and

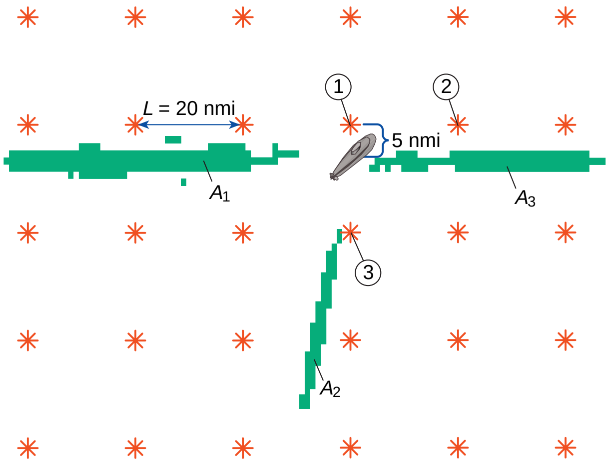

Figure 4. Effective receiver area for three sources, $A_{\mathrm{R}} = A_1 \cup A_2 \cup A_3$.

φ_H denotes the target heading angle, a uniformly distributed random variable. The angular brackets stand for averaging, first over target headings for fixed locations $\vec{x}$ then over the locations. Figure 5 shows the effective receiver area (from Eq. 4) for nine sources. That is, M has been cut off at M = 9, the source in the unit cell containing the target plus 8 nearest sources in a square grid. Calculations were facilitated by choosing sample points for location, uniformly spread over the unit cell. Ten sample points were spaced over one-eighth of the unit cell, taking advantage of the inherent symmetry in the source configuration. The distribution of target headings was also treated by sampling; in this calculation 15° steps were used. Figure 5 was computed by varying the source spacing L, i.e., the area per source is $1/L^2$.

With the revised, more general, version for computing effective receiver area (that can be implemented on a PC for most propagation and reverberation and receiver beam pattern codes), the relation to detection performance is, as before, given by Eq. 3.

EXAMPLES

For fixed (or desired or required) probability and number of ping cycles, Eq. 3 shows a linear relation between the effective receiver area A_R (an implicit function of area per source) and the area per receiver, the inverse of receiver density σ_R,

$$A_R = (1/m)\ln[1/(1-P_m)]\cdot 1/\sigma_R \;.$$

Therefore, many useful quantitative observations can be readily inferred by plots such as those in Figs. 6 and 7.

Consider the sensitivity to the directivity index (DI), first by comparing curves of A_R for different values of DI. Figure 6 shows three such curves, one for DI = 10 dB (“reference” value) and the others for DI = 5 and 15 dB. For DI = 15 dB, A_R is about 3 times as great as for the reference value, while for DI = 5 dB it is about half as large. This directly translates into the number of receivers needed by a simple construction shown on the figure: to reach probability 0.9 in 4 ping cycles in an area 10,000 nmi^2 with 10 sources requires 29 receivers for the reference case, 10 receivers if there is a 5-dB increase in DI, and 58 receivers if there is a 5-dB decrease. This results primarily from the fact that increasing the DI leads to significant reduction in reverberation. For the portion of effective receiver area that is dominated by reverberation (rather than ambient noise), reducing reverberation translates directly to relaxing the TS requirements. The noise-dominated portion of the effective receiver area is also enlarged by increasing DI, causing effective regions to be found at greater distances from the target.

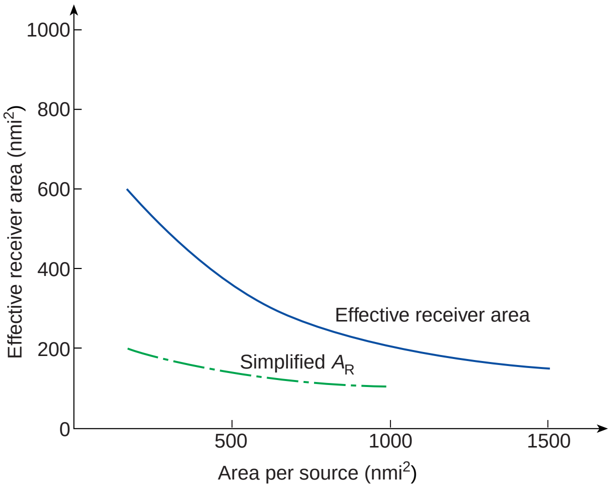

Figure 5. Example of effective receiver area plotted against area per source (nine nearest-neighbor sources, reverberation included).

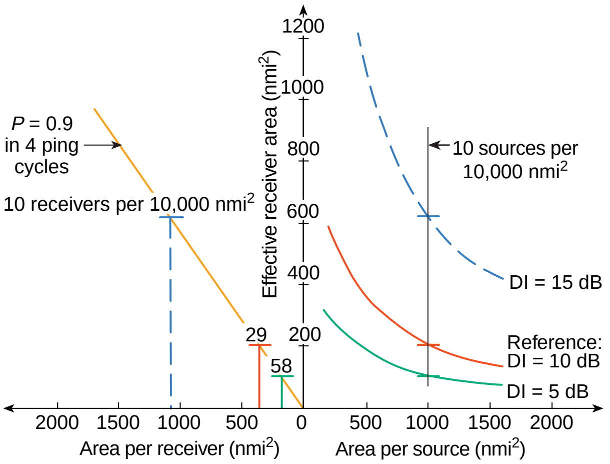

Figure 6. Sensitivity to directivity index (DI).

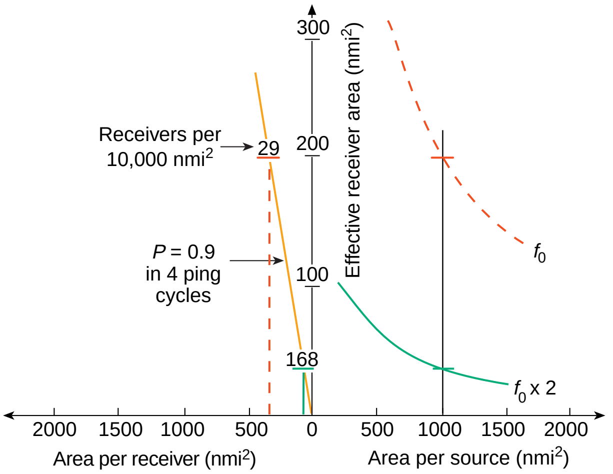

Figure 7. Sensitivity to frequency f.

For another example, consider that the (center) frequency f_0 of the multistatic system is doubled to $2f_0$. As shown in Fig. 7, this causes a pronounced reduction in the effective receiver area. The reason lies in input data: In this environment, the so-called bottom scattering coefficient, for sound backscattered from the bottom, increases sharply with frequency. This is the dominant source of reverberation over the frequency band of the impulsive sources used in the DARPA program. As indicated in Fig. 6, about 6 times as many receivers would be needed at the higher frequency (assuming, as in this calculation, no change in DI), despite the measured ambient noise being considerably less at the higher frequency.

SUMMARY

The MPPM facilitates estimating the number of sources and receivers needed to provide detection performance levels (probability) in time (number of ping cycles).

The core of MPPM is the effective receiver area. This is a construct obtained by calculating that area within which a receiver, if present, would detect a submarine at a specified position and heading when insonified by sources uniformly spread over the search area, then suitably averaging over position and heading.

The basic calculation in evaluating the effective receiver area is the active sonar equation. Hence, any computational technique that yields a SNR can be used for this evaluation, such as those that do so when embedded within an operational Monte Carlo simulation.

Examples were presented to illustrate the concept, the calculations, and some applications of MPPM. These examples were for impulsive sources because DARPA's interest in the methodology was stimulated by measurements with simple impulsive sources. The methodology is not limited to such sources; in particular, it can be applied as well to Doppler-sensitive sources. Likewise, development of the methodology was illustrated here using a detection model that results in contact if and only if $SE > 0$. The methodology also can be applied to other detection models, in particular, letting the detection probability be determined by a distribution function, with the argument a function of computed SNR.

ACKNOWLEDGMENTS: We thank Dr. William Carey, now at the Naval Undersea Warfare Center, for his interest, encouragement, and support; Carl A. Mauro of D. H. Wagner Associates, Inc., for adding the programs to SRAPS (Surveillance Resource Analysis and Planning Systems) that facilitate the effective area computations used in MPPM; and Tina M. Higgins of the APL Information Systems and Technology Group for thoroughness and diligence in providing the calculations used in the illustrative examples.

THE AUTHORS

JULIUS I. BOWEN is a Principal Professional Staff physicist in the Information Technology Group of the Submarine Technology Department, which he joined in 1980. He has B.S. and M.S. degrees in physics from Syracuse University and a Ph.D. in physics from Catholic University. Before he came to APL, Dr. Bowen was engaged in development at the Naval Ordnance Laboratory and in communication theory research at the Bell Telephone Laboratories; he then conducted and managed systems performance analysis at Raff Associates, Inc., and ORI, Inc. At APL he has concentrated primarily on effectiveness analysis and predictions for underwater acoustic detection systems. His e-mail address is julius.bowen@jhuapl.edu.

RONALD W. MITNICK is a member of APL's Principal Professional Staff and the Program Manager for Information Systems in the Maritime Technologies Program Area of the Submarine Technology Department. He has a B.A. degree in physics from Yeshiva College. Before joining APL in 1991, he was a Program Manager for the Naval Space and Warfare Systems Command, where he led efforts to develop and field advanced undersea surveillance systems. At APL, he has concentrated primarily on concept development for advanced tactical antisubmarine warfare and maritime surveillance. His e-mail address is ron.mitnick@jhuapl.edu.