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THE EVOLUTION OF EARTH GRAVITATIONAL MODELS USED IN ASTRODYNAMICS

Earth gravitational models derived from the earliest ground-based tracking systems used for Sputnik and the Transit Navy Navigation Satellite System have evolved to models that use data from the Joint United States–French Ocean Topography Experiment Satellite (Topex/Poseidon) and the Global Positioning System of satellites. This article summarizes the history of the tracking and instrumentation systems used, discusses the limitations and constraints of these systems, and reviews past and current techniques for estimating gravity and processing large batches of diverse data types. Current models continue to be improved; the latest model improvements and plans for future systems are discussed. Contemporary gravitational models used within the astrodynamics community are described, and their performance is compared numerically. The use of these models for solid Earth geophysics, space geophysics, oceanography, geology, and related Earth science disciplines becomes particularly attractive as the statistical confidence of the models improves and as the models are validated over certain spatial resolutions of the geodetic spectrum.

INTRODUCTION

Before the development of satellite technology, the techniques used to observe the Earth's gravitational field were restricted to terrestrial gravimetry. Measurements of gravity were adequate only over sparse areas of the world. Moreover, because gravity profiles over the oceans were inadequate, the gravity field could not be meaningfully estimated.

Satellite tracking technology changed all that. Observations of gravity's effect on satellite orbits, along with new and sophisticated processing techniques, provided the opportunity to obtain precise knowledge of the gravity field. However, even in the not-so-distant past, discrepancies existed among the various gravity model solutions used within the astrodynamics and geodetic communities; no general model could be used with equal confidence for orbit determination and geodetic purposes. Two primary reasons for this lack were the absence of observational coverage over the full spectral range and the many models tailored for specific satellite orbits. By incorporating common attributes and components in today's models, model developers have eliminated many of the discrepancies. This trend will continue as we better understand the detailed structure of the global gravity field and the limits of ground-based instrumentation systems in supplying the basic knowledge needed for the low- and medium-frequency portions of the spectrum.

Sputnik I, the first artificial satellite, was launched into Earth orbit in the fall of 1957. Less than 90 days later, it decayed into the atmosphere. A month later, Sputnik II followed; it lasted about 6 months. In 1958, Vanguard 1, Explorer 1, and Sputnik III were launched; Sputnik III decayed after 11 months in orbit. In 1959, Vanguard 2 and 3 and Explorer 7 were placed in orbit. Thus, by the beginning of 1960, eight satellites had been launched into Earth orbit. Of these, five were still orbiting the Earth when the satellites of the Transit Navy Navigational Satellite System (NNSS) were launched starting in 1960. The Sputniks were all launched into near-critical orbit inclinations of about 65°. (The critical inclination is defined as that inclination, $I = 63°26'$, where gravitational perturbations do not move the perigee.) Vanguard and Explorer were launched into 33° and 50° orbit inclinations, respectively; the Transit satellites went into near-polar orbits. This handful of satellites provided enough tracking and observational information to be useful for estimating the low-degree zonal coefficients of the Earth's gravitational potential.

These early satellites motivated an intense and continuing study of the Earth's gravity field using satellite tracking data. Figure 1 shows time windows for some of the major space programs from 1957 to the present. Satellites used for geodetic analysis and gravitational field modeling and estimation have included ANNA (Army/Navy/NASA/Air Force), Echo, Pageos, GEOS (Geodynamics Experimental Ocean Satellites), SECOR (Sequential Correlation of Range), Lageos (Laser Geodynamics Satellite), Geosat, Transit, Oscar, Nova, Seasat, and Topex. Figure 2 shows the navigation and geodetic satellites built by APL and used in the development of gravitational field models from the early days of satellite geodesy to today's sophisticated models.

Standard gravity models in the late 1950s and early 1960s used for various satellite programs, including Projects Mercury, Gemini, and Apollo, consisted of a hodgepodge of gravity coefficient estimates, largely because observations available for analysis were sparse, measurement systems were inaccurate, and the computer programs used in postprocessing data were rudimentary.

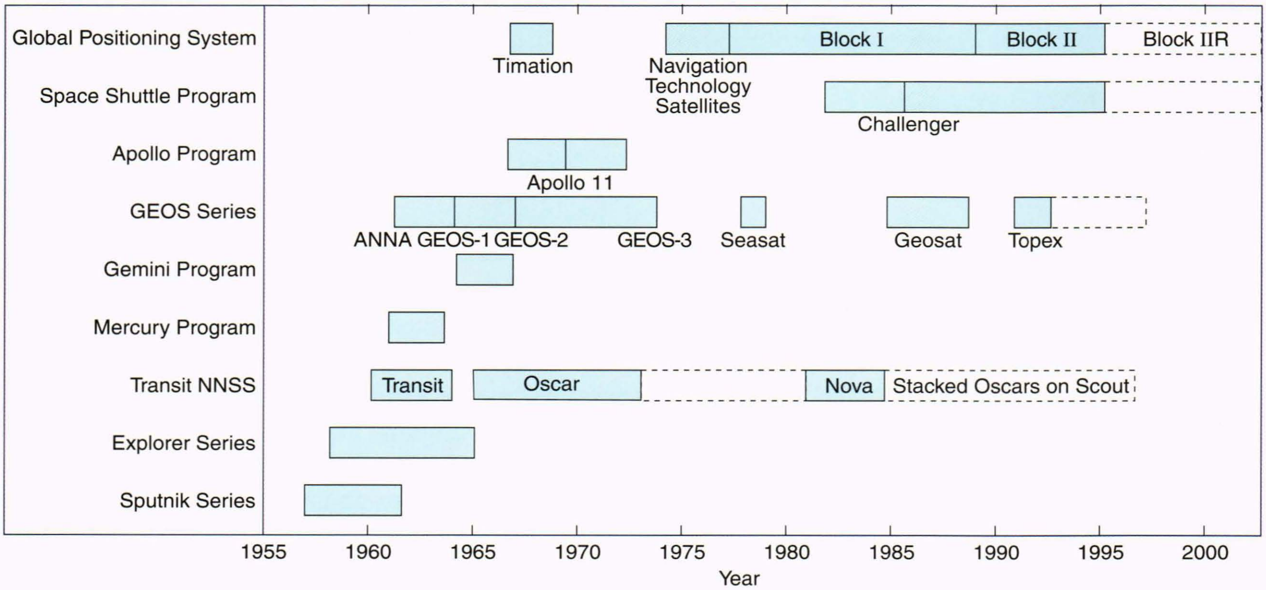

Figure 1. Historical summary of major space programs. Satellites of the Sputnik, Explorer, Transit, and GEOS series have been used for gravity field estimation.

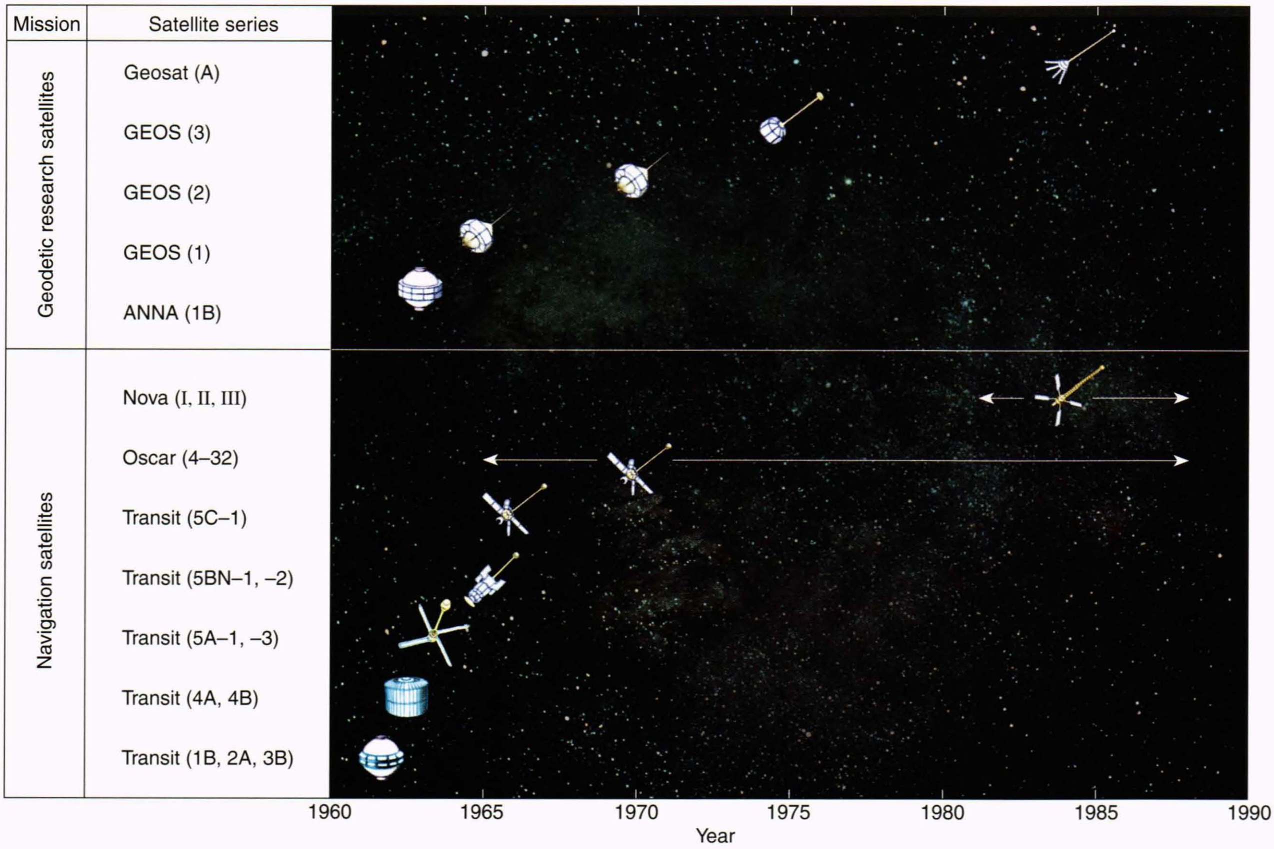

Figure 2. APL-built satellites of the Transit, Oscar, ANNA, GEOS, Nova, and Geosat series used in gravitational field estimation studies.

By the late 1960s, APL had developed a fairly complete gravity model for the Department of Defense (DoD) community based on the analysis of Doppler measurements from the Opnet (Operational Network) and the Tranet (Transit Network). The last complete gravity model developed at the Laboratory from Doppler data was APL5.0, published in 1972 (Yionoulis[1]). In the mid to late 1970s and early 1980s, developments for characterizing the models used in integrating the equations of motion grew much more sophisticated, evolving from early IBM 7094 mainframe computers to the Cray supercomputer in today's postprocessing environment. As technology creates more efficient workstations that can handle parallel processing and multitasking operations, the trend will continue toward workstation processing environments. Tracking systems have likewise continued to improve, until today we have highly precise and accurate measurement systems that are largely devoid of the systematic errors which plagued earlier systems.

Determining the fine structure of the gravity field to an accuracy level of a few centimeters is important for many fields of geophysics, from crustal structure to ocean currents. Figure 3 shows the evolution of gravity models over the last 30 years. Not until the Smithsonian Astrophysical Observatory (SAO) published the first standard Earth gravity model in 1966 did the astrodynamics community receive an unclassified gravity field description complete to degree and order 8. This model used purely Baker–Nunn camera observations. In 1964, NASA established the goal of a National Geodetic Satellite Program. The Goddard Space Flight Center (GSFC) embarked on an ambitious program to refine the description of the Earth's gravity field, which culminated in the development of a series of refined models. A complete report on the techniques and results of the key organizations involved in the first decade of the National Geodetic Satellite Program was published in 1977.[2] Over the years, many agencies and universities participated in the development of refined gravity models. The ultimate goal, however, has been to establish a unified datum and World Geodetic System (WGS) accurate to ±10 cm, which would standardize definitions of the gravitational field, the Earth reference ellipsoid, and the global geoid through the mechanism of satellite orbital analysis.

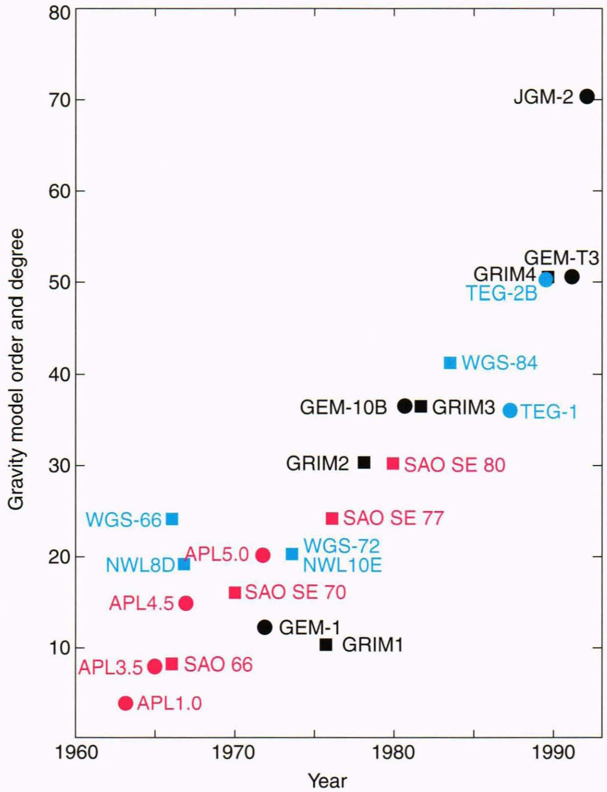

Figure 3. Evolution of gravity models. (Model series names refer to the Applied Physics Laboratory [APL]; the Naval Weapons Laboratory [NWL]; the Smithsonian Astrophysical Observatory [SAO]; the SAO Standard Earth Models [SAO SE]; the World Geodetic System [WGS]; the Goddard Earth Models [GEM and JGM]; the University of Texas [TEG]; and the GeoForschungsZentrum Potsdam and the Groupe de Recherchés de Geodesie Spatiale [GRIM]) .

The launch of the Topex/Poseidon Ocean Topography Experiment satellite and the production of a complete gravity model to degree and order 70, which combines the results and resources of diverse agencies, have brought us closer to the goal. This Joint Gravity Model, called JGM-2, was developed by NASA/GSFC, the University of Texas (UT) Center for Space Research, Ohio State University (OSU), and the Centre National d'Etudes Spatiale (CNES). It combines into one global model Doppler, laser, optical, satellite altimeter, satellite-to-satellite tracking, and surface gravity measurement data provided by NASA, UT, OSU, and the Europeans. It represents the most complete reference gravity model to date.

The main geophysical interest in constructing a detailed knowledge of the gravity field is in how the field reflects the Earth's departure from an equilibrium configuration. For example, low-degree gravity harmonics result from density anomalies and variations in the Earth's upper mantle and crust, a region less than 100 km deep called the lithosphere. Knowledge of these low-degree harmonics is essential for understanding such dynamic processes as convection currents in the Earth's core and mantle, which have been connected with large-scale plate tectonic motions. Since the higher-degree harmonics are thought to have a completely different origin from the low-degree terms, interest in defining a gravity model to high degree and order with statistical confidence will provide a useful way to check candidate convection models. Gravity field estimation shares a common theoretical basis and structure with the fields of geodesy and geophysics. Continued improvement in knowledge of the gravity field is essential to a better understanding of geophysical problems, including the Earth's geological origins and history.

This article summarizes the history of Earth gravitational field models and discusses their relevance to geodetic and geophysical disciplines. Several of the latest gravitational models are compared numerically, and their relevance for different astrodynamical studies and applications in the 1990s is assessed and evaluated quantitatively. (For a more detailed discussion, see, for example, Refs. 3 and 4.) The ultimate goal in the development of Earth gravitational field models is to allow construction of a very accurate satellite-based global model to very high degree and order (>300) that would allow detection and identification to spatial resolutions of 100 km or less. Such a model will require sophisticated onboard satellite instrumentation systems that are not yet available. For example, NASA has proposed satellite gravity gradiometers and laser- and Doppler-based satellite-to-satellite tracking systems for use by the turn of the century.

THE PRESATELLITE ERA

Sir Isaac Newton first estimated the degree of departure of the Earth's shape from a sphere. Assuming that the Earth was originally in a fluid state, Newton conjectured that the combination of gravitational and rotational effects would produce an oblate spheroid (*Principia Mathematica*, 1686). He arrived at a value of 1/230 for the flattening, a figure he recognized to be in error due to his assumption of a uniform density for the Earth. This conclusion enabled him to offer a satisfactory explanation for the precession of the equinoxes, discovered by Hipparchus around 130 B.C., and it allowed some of the earliest estimates of the Earth's flattening.[5]

But Newton could not prove the Earth's flattened, spheroidal shape. In the 1730s when Newton's natural philosophy came to France, Jacques Cassini challenged Newton's idea. From measurements of a degree of latitude made in France, Cassini maintained just the opposite—that the Earth was flattened at the equator. Criticism that the measurement points were not far enough apart led to expeditions to Peru in 1735 and Lapland in 1736 (Mason[6]). Meanwhile, in 1740, further theoretical evidence of the Earth's oblate spheroidal shape came from MacLaurin (who also made assumptions about fluid and density). MacLaurin proved that the rate of gravity's change in latitude was proportional to the square of the sine of the latitude. Then in 1743, Clairaut (who was on the Lapland expedition) published a classic memoir giving the general equations of fluid equilibrium independent of any hypotheses regarding gravitation or density. Using Newton's inverse square law and assuming that the Earth's mass was distributed in concentric layers of uniform density, Clairaut proved that the figure of an oblate spheroid satisfied the general equations of fluid equilibrium. When the Peru expedition returned in 1744, the measurements confirmed the theory: a degree of latitude was longer in Lapland than in Peru. Through measurements of gravity at different latitudes, Clairaut's model allowed the ellipticity of the Earth to be accurately computed. His treatise was so complete that it practically closed the issue.

Even so, challenges and contributions to Clairaut's work continued. For example, Airy (1826), Callandreau (1889), and de Sitter (1924) worked on developing Clairaut's work to second order. At the turn of the century, Darwin[7] showed how the flattening could be related to the moment of inertia about the polar axis. This result, when combined with seismic data, later allowed estimates of the Earth's interior density distribution. Later in the century others—including Heiskanen (1929), Spencer-Jones (1931), and Jeffreys (1952), to name but a few—made geodetic and astrodynamical contributions to both theoretical and experimental knowledge of the Earth's shape and gravity field.

The next great strides, however, had to wait until the first rockets rumbled into space carrying artificial satellites. Satellites offered possibilities for a new, robust methodology.

BASIS OF THE EARTH GRAVITATIONAL MODEL

The fundamental expression for the Earth's gravitational potential acting on a satellite is derived as the integral solution to Laplace's equation:

$$\nabla^2 V = 0\,,$$

where the unit potential V is defined as

$$V = G \int_{\text{Volume}} \frac{dm}{r_s}\,;$$

r_s is the distance from an arbitrary incremental mass dm inside the Earth to the satellite (considered as a point mass), and G is the factor of proportionality in Newton's law of gravitation known as the gravitation constant. When r_s is formulated in terms of the vector from the center of mass of the Earth to the arbitrary mass point $\mathbf{R}$ and the satellite $\mathbf{r}$ (where R and r are their scalar magnitudes, respectively),

$$r_s = \sqrt{(\mathbf{r}-\mathbf{R})\cdot(\mathbf{r}-\mathbf{R})} = r\sqrt{1 - 2R\frac{\cos\theta}{r} + \left(\frac{R}{r}\right)^2}\,,$$

its reciprocal can be expanded to give the familiar result first obtained by Laplace, valid external to the Earth:

$$V = \frac{G}{r}\int \sum_{n=0}^{\infty} P_n(\cos\theta)\left(\frac{R}{r}\right)^n dm\,.$$

The P_n are the Legendre polynomials in $\cos\theta$, where θ is the angle between the position of the mass increment $\mathbf{R}$ and the position of the satellite $\mathbf{r}$. By converting to spherical coordinates (r, ϕ, λ) and applying Rodrigues' formula, the integrals can be evaluated, resulting in the familiar form of the Earth's geopotential, with the origin at the Earth's center of mass:

$$V = \frac{GM}{r}\left[1 + \sum_{n=2}^{\infty}\sum_{m=0}^{\infty}\left(\frac{a}{r}\right)^n P_{nm}(\sin\phi) \times (C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)\right],$$

where GM is the Earth's gravitational constant. The new terms are the associated Legendre functions of the first kind P_{nm}, the geocentric latitude ϕ and longitude λ, the (unnormalized) spherical harmonic coefficients C_{nm} and S_{nm}, and the semimajor axis of the Earth's reference ellipsoid a. More commonly this expression is written in terms of the normalized C_{nm}, S_{nm} coefficients (see Table 1). In geodetic applications, the Legendre polynomials are called zonals when they depend only on latitude ($m = 0$), sectorials when they depend only on longitude ($n = m$), and tesserals when they depend on both latitude and longitude ($n \neq m$).

Table 1. Standard form of the Earth gravitational model.

$$V = \frac{GM}{r}\left[1+\sum_{n=2}^{n_{\max}}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n}\overline{P_{n,m}}(\cos\phi)\,(\overline{C_{n,m}}\cos m\lambda+\overline{S_{n,m}}\sin m\lambda)\right]$$

Parameter	Definition
V	Gravitational unit potential function
GM	Earth's gravitational constant
r	Radius from the Earth's center of mass
a	Semimajor axis of the reference ellipsoid
n,m	Degree and order, respectively
ϕ	Geocentric latitude
λ	Geocentric longitude
$\overline{C_{n,m}}, \overline{S_{n,m}}$	= Fully normalized surface spherical harmonic coefficients
	$= \left[\frac{(n+m)!}{(2n+1)\,k\,(n-m)!}\right]^{1/2}(C_{nm}, S_{nm});\ k = \begin{matrix}1\ \mathit{for}\ m=0\\ 2\ \mathit{for}\ m\neq 0\end{matrix}$
k	$= (2-\overset{\circ}{\delta}_m)\ (\overset{\circ}{\delta}_m = 1\ \mathit{for}\ m=0 \text{ and } \overset{\circ}{\delta}_m = 0 \text{ for } m\neq 0)$
$P_{n,m}(\cos\phi)$	= Associated Legendre functions of the first kind
	$= (\cos\phi)^m \frac{d^m}{d(\cos\phi)^m}\left[P_n(\cos\phi)\right]$
$P_n(\cos\phi)$	= Legendre polynomials
	$= \frac{1}{2^n n!}\frac{d^n}{d(\cos\phi)^n}(\cos^2\phi-1)^n$
$\overline{P}_{nm}(\cos\phi)$	= Fully normalized surface spherical harmonics
	$= \left[k(2n+1)\frac{(n-m)!}{(n+m)!}\right]^{1/2} P_{nm}(\cos\phi)$

Developing the potential in terms of spherical harmonics was natural in the analysis of satellite orbits, because the symmetry properties of the harmonics correspond to the division of the potential according to the type of change in the node and argument perigee of the classical orbit elements (i.e., secular, long period, or short period). In 1961 Brouwer and Vinti recommended that the International Astronomical Union adopt the form of the potential in terms of spherical harmonics. Other forms of the potential, expressed in terms of ellipsoidal coordinates, have also been developed,[8] but they result in a more complex representation requiring use of elliptic Legendre polynomials of the second kind. In 1966 Kaula[9] developed a form of the potential in terms of the Keplerian orbit elements that was particularly well suited for analyzing gravity effects on satellite orbits.

Before the start of the space age, further efforts in understanding the Earth's geopotential focused on the associated boundary value problem, often drawing on and extending Stokes' classic paper (1849), which developed means for finding the shape of the geoid based on gravity anomalies measured at the surface. Nearly 100 years later, Meinesz (1944) extended Stokes' work to obtain the geoid shape, referring it directly to a reference ellipsoid. But the application of such techniques demanded considerable numbers of gravity measurements over vast land and ocean areas, making the process time-consuming at best. Further, the results could not provide the scale of the geoid, only linear departures from it (Rice[10]).

GEOPOTENTIAL MODELS FROM EARTH SATELLITE DATA

The potential for artificial satellites to improve knowledge of the Earth's gravitational field was recognized at least as early as 1956 (Blitzer, Weisfield, and Wheelon[11]). Variation of the potential with latitude produces changes in the satellite's orbit. Not until the first Sputniks were observed by King-Hele and his collaborators in England and by Buchar in Czechoslovakia was it found that the meridional ellipticity was notably different from that determined by surface measurements.[12] Thus by 1958 the space age had produced the first quantitative result for geodynamics: the accuracy of the second zonal harmonic (J_2) had improved from two significant figures to four.[13]

The Post-Sputnik Era (1957–1966)

During this period, Earth gravity model development consisted of individual investigators using small subsets of available satellite tracking data to estimate mostly zonal and selected tesseral harmonics of the gravity field. This era also introduced the Transit series of navigation satellites.[14,15] The Transit system concept, which was born in 1958 at APL, used two-frequency Doppler measurements to estimate the gravity field, station coordinates, frequency offset, and refraction-dependent errors. The primary application was to provide accurate position updates for the inertial navigation system onboard U.S. submarines. The first successful Transit satellite (1B) was launched in early 1960; the Oscar satellites were launched in 1965. The Transit system evolved through the Triad, TIP (Transit Improvement Program), Nova, and SOOS (Stacked Oscars on Scout) systems of satellites. Today one Oscar and one Nova satellite are operating in orbit, and the recyclable SOOS system maintains a six-satellite constellation. The Transit system will be turned off in late 1996 and replaced by the Global Positioning System (GPS). Laboratory staff who contributed notably to gravity model developments include Newton,[16,17] Guier,[18–20] Black,[21] and Yionoulis.[22] The Laboratory developed a series of models using the Orbit Improvement Program[23] starting in 1963 with the APL1.0 model, which included results to degree and order 8; the most popular was the APL3.5 model (produced in 1965), which included harmonics to degree and order 12. The final APL5.0 model (to degree and order 15) was a predecessor to the WGS-72 model published by the Defense Mapping Agency (DMA) in 1974.

The Laboratory-developed gravity models, although complete to degree and order 15, were specifically tuned to the polar orbits and altitude of the Transit system, and they were designed for military use. The tracking accuracy history for each geopotential model obtained from the Transit system over nearly a 20-year period is illustrated in Fig. 4. By the end of the 1970s, when the Laboratory stopped constructing gravity models and DMA received the DoD charter for geodesy, the total error over a typical tracking span was in the range of 5 to 10 m.

The late 1950s and early 1960s also saw a surge of activity devoted to use of suborbital rocket flights and ballistic missile flight tests over relatively short ranges. Generally for ranges less than the radius of the Earth, only the second-harmonic (J_2) was included in trajectory computations. The along-track error for neglecting J_2 ranged from 2 km at suborbital ranges (2000 km) to 20 km at longer ranges (about 6000 km). The effects of higher-order zonal harmonics and of the tesseral and sectorial harmonics were a few orders of magnitude less and were generally neglected from most early considerations of accuracy. Not until high-precision trajectory reconstruction became a requirement were higher-order gravity effects even considered for calculating best estimates of trajectory.

The decades after the first satellites were orbited saw an intense increase in the use of artificial satellite data to expand knowledge of the Earth's geopotential function. For a number of reasons, early estimates of the geopotential coefficients differed significantly from one

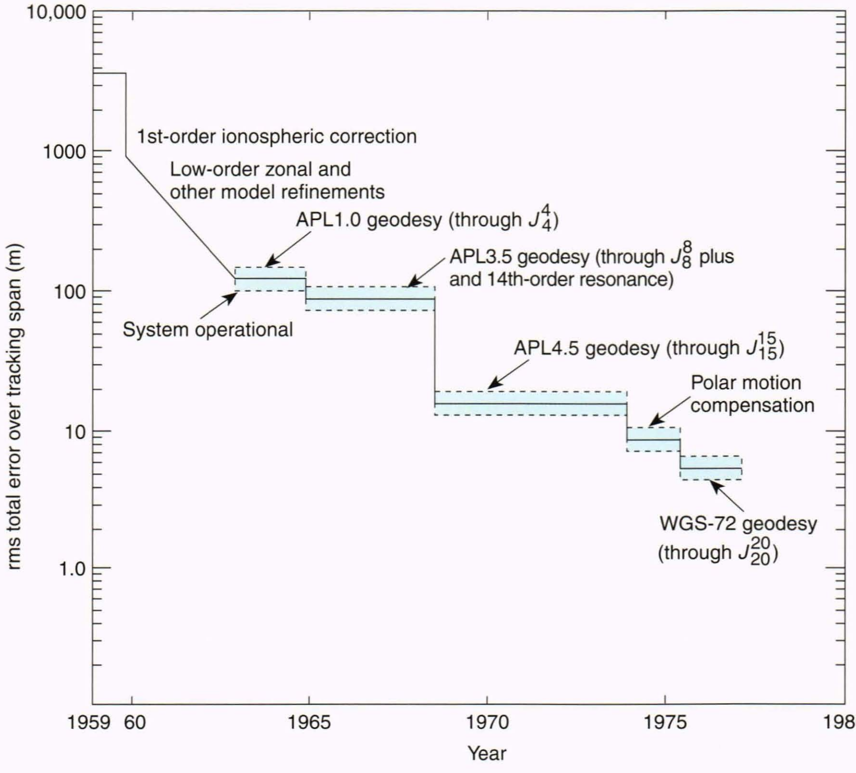

Figure 4. Tracking accuracy of geopotential models obtained from the Transit system over the 20 years of the Orbit Improvement Program.

another, and it took some time to iron out the errors. For example, at the first symposium on the topic, "The Use of Artificial Satellites for Geodesy," held at the U.S. Naval Observatory in 1962, estimates of the second zonal harmonic coefficient (J_2) ranged from 1.0822×10^{-3} to 1.0833×10^{-3}, and values for other coefficients were scattered widely.[24] These discrepancies produced concentrated activity to straighten out the disagreements, aided by the first satellite launched for geodetic purposes, ANNA 1B (October 1962). By the second meeting, "enormous progress" had been made, with J_2 estimates improving 2 orders of magnitude.

By 1964, 7 years after the launch of Sputnik I, with more than 50 satellites launched in orbit and gravity coefficients evaluated by teams of investigators, it was fairly well recognized that the second-, third-, and fourth-degree zonal harmonics had been determined to high accuracy; however, accuracy of the higher-order zonals and all tesserals was still doubtful. Moritz[25] stated that even under ideal conditions (by no means attained at the time), "Satellite orbital analysis cannot give meaningful harmonics to higher than 10th or perhaps 20th degree at best." The reasons given were the rapid attenuation of the higher harmonics with increasing altitude, tracking system noise, satellite modeling errors, standardization of reference coordinate systems and station position errors, and measurement system calibration errors.

The Apollo Era (1966–1972)

The National Geodetic Satellite Program focused the efforts of teams of investigators from NASA, DoD, and the Department of Commerce to develop a unified world datum and provide a refined and accurate description of the gravity field. The first comprehensive model using Baker–Nunn camera data was produced by SAO[26] in 1966 and provided a complete field to degree and order 8. This led to a more refined field complete to degree and order 16 using a combination of laser and Baker–Nunn camera data in 1970. It represented the SAO Standard Earth (SE) model series, which was developed over the next decade, culminating in SAO SE 80.

Beginning in 1967 with NWL8D, the Naval Weapons Laboratory (NWL, later the Naval Surface Warfare Center) derived gravity model solutions complete to degree and order 19 using only data from the Doppler Navy Navigational Satellite System. These efforts continued through 1972 with NWL10E, which became the basis of the WGS-72 model development efforts initiated by DMA. The NWL10E model was updated in 1984 using GEOS-3 and Seasat altimeter data. The DoD completed its gravity model development effort in 1974 with WGS-72, which was derived from diverse observations but was particularly strong in satellite Doppler measurements. (An early DoD gravity model, WGS-66, was published in 1966 complete to degree and order 24; it included limited satellite and surface gravity data, although initial model development [WGS-60] contained no satellite data, and included only surface gravity and astrogeodetic data.)

The levels of accuracy obtainable with early efforts allowed various error sources to be ignored. The Smithsonian 1966 investigation included polar motion, but the effects of ocean and solid Earth tides were small enough to be ignored. Later efforts, however, required that such contributions (including relativistic effects) be accounted for. At APL, Newton[27] did some fundamental work in the mid-1970s on satellite determination of Earth tidal parameters, and Jenkins[28,29] contributed significantly by accounting for the effects of relativity on Doppler-processed orbits. In addition, new data types became available to make estimating and accounting for various error sources possible. These systems included the early Baker–Nunn cameras, which offered precision of their reduced observations at the level of 3–5 arc-sec; the early minitrack interferometric radio tracking system, which gave angular precision on the order of 20 arc-sec; and also radio ranging systems, Doppler tracking systems, satellite radar altimeters, and laser tracking systems. For example, to help determine the shape of the maritime geoid, the GEOS-1, 2, and 3 and the Seasat missions used radar altimeters to directly measure sea-surface height. Determining the shape of the maritime geoid required modeling of both the solid Earth and the ocean tides. Seasat also employed laser measurements for extremely accurate satellite range information.

The Post-Apollo Era (1972–Present)

The first Goddard Earth Model (GEM) was developed by GSFC in 1972 complete to degree and order 12. The data consisted of Baker–Nunn camera data only. In subsequent years, additional data types were implemented, including laser range, unified S-band range and range-rate, and minitrack interferometer measurements; the GEM models were published serially over nearly 20 years from GEM-1 and GEM-2 to GEM-9 and GEM-10. The odd-member series were satellite-only models. The even-member series were from combined satellite and surface gravity measurements, normally mean gravity anomalies obtained over a gridded region of the Earth. The GEM-10 model developed in 1981 included satellite radar altimetry and surface gravity measurements and was complete to degree and order 36. The next large development effort occurred with the production of the GEM-T3 model in 1992, which was complete to degree and order 50. The most recent GSFC model, due to be published in 1994, is the JGM-2 model complete to degree and order 70. It includes Topex satellite data and Doppler Orbitography Radiopositioning Integrated by Satellite (DORIS) tracking data provided by France.

In 1983, investigators at the Jet Propulsion Laboratory and the University of Texas identified the rate of secular variation in the Earth's J_2 coefficient ($J_2 = -3 \times 10^{-11}$/year) using laser tracking data for the Lageos.[30] Thus, as the desire for ever-increasing detail grew, so did the complexity of the problem, and more precise methods and instruments were required. As more harmonic coefficients were estimated, more varied and precise data were needed, along with increasingly powerful computers and numerical methods.

Investigators besides NASA, SAO, APL, NWL, and DMA who also developed gravity models in this period include the UT, OSU, and the European Space Agency (ESA). The UT models, called TEG models, were based on laser range, Doppler range-rate, and radar altimetry data over a wide distribution of inclinations (15–115°) and also included surface gravity data. A preliminary model, TEG-1, was released in 1988; a refined solution to degree and order 50 (referred to as TEG-2B) was published in 1990; and TEG-3, which is due in 1994, will include Topex data. The European model development effort was begun in 1976 with GRIM1, which used only laser and optical data. The most recent model (GRIM4B), published in 1990, included satellite laser range, optical, and DORIS tracking data and 1 × 1° surface gravity data provided by OSU.

The OSU models comprise very-high-order models up to degree and order 360. They are determined starting with a previously determined gravity field. Cook[31] used surface gravity data in combination with satellite data as early as 1958, but such combinations were first seriously used in 1961 by Kaula[32] (with minitrack data) and then in 1968 by Rapp, who used them in early models at OSU. The early databases were sparse but have become denser as additional measurements are obtained and added to the data bank. One such database exists at OSU and another at the DMA Aerospace Center in St. Louis, Missouri, for use in WGS model developments (e.g., WGS-72 and WGS-84). The first OSU model (OSU86) was published in 1978: it used 5 × 5° gravity anomalies averaged from the 1 × 1° databases and the GEM-9 field, and it was complete to degree and order 180. Model OSU89B, published in 1990, started with the GEM-T2 field. It used 30 arc-min surface gravity anomalies from the OSU data bank and GEOS-3/Seasat altimeter-derived gravity anomalies. The most recent model, OSU91A, used an updated database and was published in 1991 complete to degree and order 360.

When the DoD WGS-72 gravity model was developed in 1974, it was claimed as the largest collection of data ever used for world geodetic purposes. In 1986 this model was superseded by WGS-84,[33,34] which is the standard reference used by the U.S. military and DoD agencies. The entire GPS, for example, is based on the WGS-84 system, even though much better models exist today outside the defense community. Above a certain degree and order, all DoD models of the gravity field were classified until late 1993, including the APL, NWL, and WGS models. Recently, WGS-84 was declassified for the complete field. Previous versions of such models as WGS-72 and WGS-84 were classified beginning above a higher-order and degree field only (12 and 41, respectively), with the lower field being unclassified. The data used in construction of the major gravity models are shown in Table 2.

TRACKING AND INSTRUMENTATION SYSTEMS

The accuracy of the geopotential models is a direct function of the instrumentation systems employed and their inherent precision. Early tracking systems were subject to various systematic errors that complicated the estimation process. As higher-accuracy systems are developed, systematic effects are being gradually minimized or even eliminated. Various tracking systems have been used. Kinetheodolite, visual observations, minitrack optical interferometer, and Baker–Nunn camera data were employed in the early period of gravity model estimation. Later systems have evolved toward electronic Doppler, unified S-band, laser, radar altimeter, and active and passive ranging systems. Table 3 lists all tracking, instrumentation, and observations used in the construction of the various gravity coefficient estimates and the recovery of full gravitational models.

ESTIMATION CODES AND SOFTWARE

Estimates of zonal gravity coefficients in the early post-Sputnik era employed the construction of coupled sets of linear equations with solved-for gravity parameters

Table 2. Data types used in gravity model developments. (ALT = altimeter; D = Doppler; L = laser; MFA = mean free air anomaly; MGA = mean gravity anomaly; MT = minitrack interferometer; O = optical; S = SECOR; SST = satellite-to-satellite tracking; USB = unified S-band.)

Model	Measurement data used	Surface gravity size (deg)	Model	Measurement data used	Surface gravity size (deg)
WGS-66	O,S,D	5 × 5	GEM-1	O	None
WGS-72	O,S,D	10 × 10	GEM-3	O,D,USB,L,MT	None
WGS-84	D,L,ALT,GPS	1 × 1	GEM-5	O,S,USB,L,MT	None
APL1.0	D	None	GEM-10B	O,D,USB,L,MT,ALT	1 × 1 MGA
APL3.5	D	None	GEM-T3	O,D,L,ALT	1 × 1 MGA
APL4.5	D	None	JGM-2	O,L,D,ALT	1 × 1 MGA
NWL8	D	None	GRIM1	L,O	None
NWL10	D,ALT	None	GRIM3	L,O,D	1 × 1 MGA
SAO 66	O	None	GRIM4	L,O,D,DORIS	1 × 1 MGA
SAO 70	O,L	1 × 1 MFA	TEG-1	L,O,D,ALT	1 × 1 MGA
SAO 77	L	None	TEG-2	L,D,ALT,USB,DORIS	1 × 1 MGA
SAO 80	O,L,ALT	1 × 1 MFA	OSU91A	L,O,D,USB,ALT,SST	1 × 1/0.5 × 0.5 MGA

Table 3. Instrumentation systems for gravity field analysis.

Instrumentation system	Measurement	Instrument precision	Time period in use
Kinetheodolites	Optical	0.5–3.0 arc-min	1957–1981
Visual observations	Optical	2 arc-min	1960–1979
SAO Baker–Nunn camera	Optical	2 arc-sec	1957–1974
Hewitt camera	Optical	1 arc-sec	1961–1985
NASA minitrack interferometer	Direction cosines	1–2 arc-min	1957–1970
Unified S-band	Average range-rate	10.5 cm/s	1966–1980
Transit/Oscar	Doppler	14 cm/s	1960–present
Nova	Charge-coupled device	4.5 cm/s	1981-present
SECOR	Passive range	2 m	1962–1965
Satellite-to-satellite tracking	Doppler	5 cm/s	1980–1985
Laser	Range	1 m (1968)	1964–1985
		5 cm (1993)	1985–present
Radar altimeter	Radar height	20 cm (Seasat)	1974–1975
		10 cm (Geosat)	1980–1985
		2 cm (Topex)	1992–present
Surface gravity	Gravity anomaly	30 mgal[a] (1960)	1960–1970
		20 mgal (1970)	1970–1980
		10 mgal (1980)	1980–1990
		3.5 mgal (1993)	1990–present

[a] 1 milligal = 0.0098 m/s^2

truncated to some relatively low degree. These estimates were based on observed changes in the nodal crossing of the orbit and the advance in the argument of perigee. When entire models were estimated, this practice was replaced by solutions of sets of normal equations for each data type, which were linked to the evolution of the dynamical orbit in space. These methods were all based on batch-weighted least-squares estimation methodology; their use continues even in today's high-order gravity recovery operations.

The early theories were based on analytic methods of solution developed largely by Brouwer and Clemence (1961).[35] Since analytical solutions were limited, approximations were continually made to accommodate such forces as drag and radiation pressure. The obvious approach was use of a Cowell method of numerically integrating the variational equations of motion to keep the errors caused by these forces from aliasing into the solutions for the gravity coefficient estimates themselves. Thus, as the order of the field, the amount of data, and the types of data increased, mainframe and then Cray supercomputers were required to perform the integrations. Recently, however, trends toward use of semi-analytic methods in a workstation environment have surfaced. These methods can accommodate the larger-dimensional gravity models complete to degree and order 50 and have been successfully compared with Cowell numerical integrators to high accuracy.[36] So far, the workstation-based, semi-analytic technique has been used only for mission support operations, but the gravity model problems will soon be addressed in an environment of this type, with parallel tasks supported by many cross-linked and multitasked workstations.

Table 4 describes the various estimation codes used for orbit determination and gravitational analysis since the initial programs were developed beginning in the early 1960s at facilities such as The Royal Aircraft Establishment in the United Kingdom, APL, and SAO.

Some of the early differential correction algorithms used in estimating geodetic parameters employed special perturbation theories to compute satellite orbits; these were somewhat faster for the computer systems available at the time. One example was the extensive work of investigators at SAO, which resulted in the Smithsonian Institution Standard Earth Model published in 1966. Twelve Baker–Nunn camera systems provided satellite position data against the star background to a precision of 2 arc-sec. This work employed both a dynamic and a geometric method of processing. The basic dynamic approach for differential correction was, in principle, the same as that used today, with the corresponding largest normal matrix being 98 × 98. This method was augmented by a geometric approach that derived the direction from one Baker–Nunn station to another directly from the data; the geometric approach was useful as an alternative technique, providing more robust estimates of the station locations. Results from this SAO model were compared with surface gravity measurements and indicated agreement to within 10 milligal (1 milligal = 0.0098 m/s^2). Uncertainty in the station locations was on the order of 15 m.

A combination-based approach using satellite tracking data, altimeter data, and surface gravity measurement data provides better insight into subtle data incompatibilities between the systems. The data mix reflects widely varying accuracies and spectral sensitivities to the

Table 4. Estimation software used in gravity analysis.

Year	Agency	Developer	Software
1959–present	U.S. Navy	APL	Orbital Improvement Program (OIP)
1963–1980	SAO	SAO	Differential Orbit Improvement (DOI)
1967–1985	Royal Aircraft Estabishment	United Kingdom	Program for Orbital Parameters (PROP)
1976–present	GeoForschungsZentrum Postdam and Groupe de Recherchés de Geodesie Spatiale	Germany/France	German Processing and Archives Facility
1980–present	U.S. Navy and DMA	Naval Weapons Laboratory/ Naval Surface Weapons Center; DMA	Celest
1980–present	NASA/GSFC	Wolf R&D Corp.	Geodyn I Geodyn II
1980–present	University of Texas	UT Center for Space Research	University of Texas Orbit Procesor (UTOPIA)

gravitational signal. Today's solutions are highly computationally driven; they require rigorous statistical and numerical techniques to optimally combine the various data types and produce realistic accuracy estimates. For example, when using surface gravimetric data, investigators commonly observe aliasing effects in the computational model caused by field truncation and datum and vertical reference system problems. Some of these problems are aggravated by the nonuniform quality and coverage in the surface gravity data themselves, resulting in a lack of long-wavelength integrity caused by unknown systematic errors. The surface gravity data, however, play a fundamental role in defining the short-wavelength content of the field.

The WGS-72 gravity field estimates were done in the early 1970s to degree and order 20 on mainframe IBM 7094/360 vintage computers that required some 500 terms in the geopotential to be estimated; the most recent solutions for JGM-2 complete to degree and order 70 were done on Cray supercomputers and required more than 7000 terms to be estimated. In addition, to attain the centimeter-level accuracies required by the Topex spacecraft, the orbit characteristics must be precisely modeled to handle nonconservative orbit forces such as radiation pressure, atmospheric drag, thermal imbalances, and spacecraft emissions effects. In 1960, the computation of the gravity potential required 50 h of IBM 704 data processing time using only a handful of satellites and 10 tracking stations from the minitrack radio interferometer tracking network to obtain a complete solution. In contrast, current supercomputer operations performed with parallel processing require 2 h on a Cray central processing unit to obtain a complete solution for the JGM-2 70,70 gravity field based on use of combined multi-arc satellite solutions.

REVIEW OF GRAVITY MODELS IN CURRENT USE

The following paragraphs briefly describe seven of the latest and most popular gravitational models. These models are employed by a wide spectrum of users for the various applications.

OSU91A—This model was computed by combining the GEM-T2 gravity model with satellite altimeter data and surface gravity data. It was produced by OSU and is complete to degree and order 360.[37]

TEG-2B—This model was computed from a combination of satellite tracking data (laser ranging, Doppler range-rate and satellite-to-ocean radar altimeter) and surface gravity data provided by OSU. It was produced by the UT Center for Space Research and is complete to degree and order 50. Details of the model have been presented at numerous scientific meetings, but no formal journal article documenting the model has yet appeared.

GRIM4-C3—The GRIM4 series of global Earth gravity field models was developed within a German–French cooperation between the Institutes GeoForschungsZentrum Potsdam (Potsdam, Germany) and Groupe de Recherchés de Geodesie Spatiale (Toulouse, France). The GRIM4 models exist as satellite-only versions (GRIM4-S) and as combined solutions (GRIM4-C). The GRIM4-S series is derived from optical, laser, and Doppler tracking data of some 30 satellites, with gravitational coefficients complete up to degree and order 50 and some resonant terms up to a maximum degree of 66. The GRIM4-C models combine the satellite-only normal equations with surface data (gravity anomalies and altimeter-derived geoid undulations); these models have complete coefficients up to degree and order 60, corresponding to a 350-km half-wavelength resolution on ground. The GRIM4-S models are designed primarily for use in precise orbit determination. They support the Earth Resources Satellite (ERS-1) and Satellite pour l'Observation de le Terre (SPOT), developed by the ESA and the joint NASA–Centre National d'Etudes Spatiale (CNES) Topex/Poseidon missions. The GRIM4-C solutions are state-of-the-art representations of the long-wavelength geoid. Both lines of models are used extensively in the ERS-1 processing and archiving facilities for precise orbit restitution and for altimeter data reduction. The most recent versions (1993) are GRIM4-S4 and GRIM4-C3 for satellite-only and for combined solutions, respectively.[38,39]

WGS-84—This model, the latest DoD model computed by the DMA, was recently declassified. It is complete to degree and order 180 and was computed from a combination of satellite tracking data (10 satellites), satellite altimeter data (Seasat and GEOS-3), and DMA Aerospace Center surface gravity data.

GEM-T3—This model is part of a series (T1, T2, T3) produced by the Space Geodesy Branch at GSFC to support orbit determination for the Topex/Poseidon mission.[40–42] It is based on satellite tracking (31 satellites), satellite altimeter data (GEOS-3, Seasat, Geosat), and surface gravity data. Although this series of gravity models was developed for Topex,[43,44] the models are in general use and are not biased toward Topex. The error covariances of the models have been extensively calibrated to realistically represent the errors in each model. The surface gravity data were processed and supplied in the form of normal equations by OSU.[45]

JGM-1—This model, called the Joint Gravity Model (also in memory of the late J. G. Marsh), is the final Topex/Poseidon prelaunch gravity model. It was produced in a collaboration between GSFC, CNES, OSU, and the Center for Space Research at UT. The model represents a complete reiteration of the data contained in GEM-T3 with many improvements in the background models (e.g., International Earth Rotation Service constants were used wherever possible). It is complete to degree 70. The model was significantly improved for satellites in a Sun-synchronous orbit through the addition of SPOT-2 DORIS tracking data from France.[46] Because the altimeter data and surface gravity data were smoothed using the OSU91A model, that model can be used to extend JGM-1 from degree 71 to 360.

JGM-2—This model is identical to JGM-1 except that it includes satellite laser range (SLR) and DORIS tracking data from Topex/Poseidon. It is the final Topex postlaunch gravity model.[47] Besides GSFC, the Center for Space Research at UT, OSU, and the CNES participated in its development, as was done for JGM-1. This model is complete to degree 360 using the OSU91A model. Currently, JGM-2 represents the best available long-wavelength model of the Earth; nevertheless, a number of improvements are planned over the next few years. A formal publication of JGM-2 is expected as a special issue of the *Journal of Geophysical Research* in 1994.[48]

COMPARISON OF CONTEMPORARY GRAVITY MODELS

Gravitational models can be evaluated using various methods including orbital fits, recovery of the surface anomaly field, and altimetric geoid. This section provides results recently completed based on Geodyn simulations using the seven contemporary gravitational models described, all of which are used within the astrodynamics community. The results of these performance estimates and comparisons were presented at the 18th General Assembly of the European Geophysical Union.[49] Table 5 lists the models tested, along with the data elements each model uses.

Although many data sets are common to each model, the models have significant differences, which depend on several factors: the number of satellites used in the solution, the terrestrial gravity and altimeter data used, the editing criteria employed, the relative weights applied to the different data sets, the estimation techniques used, and the number and type of parameters estimated. Given the many different data types used in generating these models, independent data are not abundantly available to independently test the accuracy of these models. The following tests were used to compare the different models:

1. Least-squares fits to different sets of satellite tracking data
2. Comparison of Topex orbits, derived using SLR/DORIS tracking data plus a given gravity model, with independent GPS-derived orbits
3. Comparison with geoid undulations implied by Doppler positioning
4. Comparison with GPS-leveling-derived geoid undulations
5. Comparison with Topex-derived geoid undulations

The geoid differences between the models evaluated are shown in Table 6. Generally the differences to degree and order 50 cluster around 50 cm rms, except with WGS-84, which is clearly an outlier; its rms differences from the other models are greater than 1 m. Figure 5 shows the observation residuals resulting from fitting an orbit to the satellite laser ranging data using the various gravity models. Results are provided for Lageos I, Lageos II, Starlette, and Ajisai, which have orbit altitudes of 5900, 5900, 950, and 1500 km, respectively. Although orbit mismodeling occurs for reasons other than gravity errors, the fits generally show the level at which each gravity model represents the gravitational perturbations experienced by the particular satellite. The Lageos II test is particularly good because the nonconservative forces are small and none of the tested models contains Lageos II tracking data.[50] Clearly, WGS-84's performance is the worst. Performance of the other models is similar, although JGM-2 is best. Figure 6 demonstrates how each model performs for Topex orbit determination. For each gravity model, an orbit was computed using SLR and

Table 5. Comparison data for current gravity models.

Gravity model	Number of satellites	Satellite data types[a]	Perigee altitude (km)	Inclination (deg)	Radar altimeter	Surface gravity data[b]
WGS-84	10	Tranet Doppler Laser range Pseudorange (GPS)	800–1,100 5,700 (Lageos) 20,500 (GPS)	50–90	Seasat GEOS-3	1 × 1° MGA (DMA) 10^6 point anomalies (Naval Oceanographic Office)
GEM-T3	30	Tranet Doppler Optical Laser range USB avg. RR SST	600–2,000 35,000 (ATS6)[c] 5,900 (Lageos)	1–144	Geosat Seasat GEOS-3	1 × 1° MGA (OSU)
JGM-1		Tranet Doppler Optical Laser range USB avg. RR SST DORIS/SPOT2	600–2,000 35,000 (ATS6) 5,900 (Lageos)	1–144	Geosat Seasat GEOS-3 DORIS/SPOT2	1 × 1° MGA (OSU)
JGM-2		Tranet Doppler Optical Laser range USB avg. RR SST SLR/Topex DORIS/Topex	600–2,000 35,000 (ATS6) 5,900 (Lageos)	1–144	Geosat Seasat GEOS-3 DORIS/SPOT2 DORIS/Topex	1 × 1° MGA (OSU)
TEG-2B	30	Laser range USB RR Tranet Doppler	780–1,600 5,900 (Lageos) 19,200 (Etalon)	15–108	Seasat GEOS-3 Geosat	1 × 1° MGA (OSU)
OSU91A	30	Optical Tranet Doppler Laser range USB RR SST	600–2,000	1–144	Seasat GEOS-3 GEOS-2 Geosat	1 × 1° MGA (OSU) 30 × 30′ MGA
GRIM4-C3	32	Optical Laser range Tranet Doppler	19,200 (Etalon)	15–115	DORIS/SPOT2	1 × 1° MGA (OSU)

[a] RR = range-rate; SST = satellite-to-satellite tracking; USB = unified S-band.

[b] MGA = mean gravity anomoly.

[c] ATS6 = Advanced Technology Satellite 6.

Table 6. Matrix showing geoid differences for different gravity models.

	rms differences between models (global/land/ocean) (cm)					
Model	OSU91A	GEM-T3	JGM-1	JGM-2	TEG-2B	GRIM4-C3
GEM-T3	48/71/34					
JGM-1	43/64/30	55/84/38				
JGM-2	43/65/30	56/84/38	4/6/3			
TEG-2B	56/91/32	49/70/37	50/84/26	50/83/26		
GRIM4-C3	62/90/46	63/96/43	59/88/43	59/88/43	64/99/42	
WGS-84	114/177/75	115/176/78	116/183/72	116/813/72	112/176/72	153/249/79

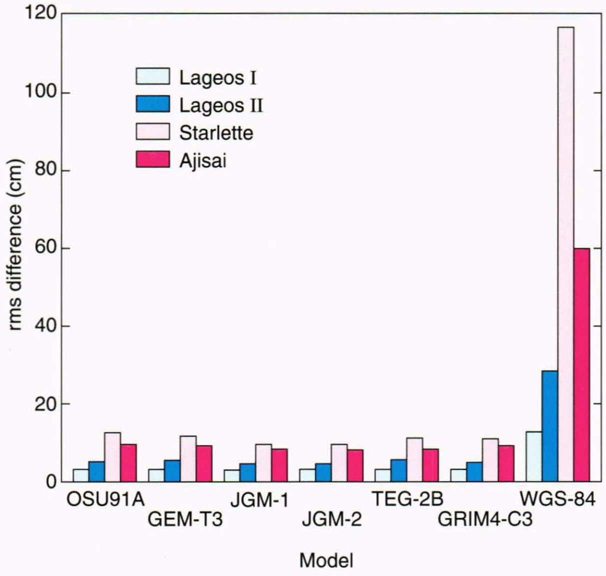

Figure 5. Satellite laser range (SLR) orbit fit differences for various gravity models.

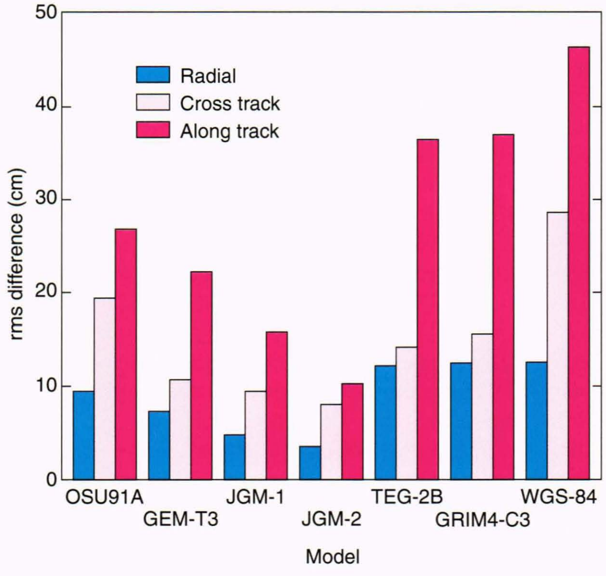

Figure 6. Topex orbits calculated with gravity models using SLR plus DORIS tracking data compared with Topex orbits computed from GPS precision ephemerides.

DORIS tracking data. Each model-computed orbit was then differenced with a precision orbit computed by the Jet Propulsion Laboratory using GPS tracking data in the "reduced-dynamic" orbit determination mode, claimed to be largely free of dynamic errors in the orbit modeling process. In this test, JGM-2 has a distinct advantage, because the model includes Topex tracking data. For the radial component, performance of WGS-84, TEG-2B, and GRIM4-C3 is similar. As expected, the JGM series models perform the best for Topex. Table 7 compares geoid undulations from each model with undulations computed from Topex altimetry. Each model was extended from its maximum degree to degree 360 using OSU91A to make the comparisons with the independent data types. The representations of the geoid from JGM-1 and JGM-2 are better than those of the other models; the comparisons to WGS-84 are the worst.

Various characteristics must be considered in selecting a gravity model for a particular application. The tests described here represent only a small subset of tests that could be developed. For precise geodetic work, JGM-2 clearly has a small advantage over the other models, with its advantage over WGS-84 being most noticeable. Where orbit accuracies of only a few meters are required, however, any of the tested models, including WGS-84, is more than sufficient. Indeed, for most astrodynamic applications, any of the models would probably provide sufficient accuracy. The best model for the long wavelengths of the gravity field (up to degree 70) is JGM-2, with extension up to degree 360 using OSU91A when high spatial resolution is required.

THE FUTURE OF GRAVITY MODEL DEVELOPMENT

Gravity models are continually being extended and results combined in an optimal solution to provide components of as short a wavelength as possible and to work toward a unified global system.

Satellite Tracking and Gravity Measurement Trends

The best combinations of satellite orbits and observations improve estimates of the long-wavelength components (low degree and order coefficients). The medium wavelengths are estimated from satellite altimetry and surface gravity measurements. Estimating the short-wavelength components with ground-based measurements, however, would require as many as 300 well-distributed ground stations that could track low satellite orbits (160 km) to obtain the best resolution of harmonics and separation of coefficients. An added complication in the use of ground-based stations is that atmospheric noise in the signal is larger than the orbital perturbations being measured and evaluated. Thus, estimates of short- and very-short-wavelength components (to degree and order of more than 100) will have to await the use of highly accurate satellite-to-satellite tracking techniques and orbiting gravity gradiometers (see Fig. 7, which shows the predicted increase in accuracy with use of orbiting gradiometers).

An orbiting gradiometer was proposed as early as 1974. The University of Maryland, APL, and the French government were among those who helped to develop the concept.[51] In 1982 NASA proposed a dedicated gravitational satellite mission for geopotential research (Gravsat), but the mission was terminated because cost estimates for postprocessing the data were excessive.

Table 7. Comparison of geoid undulations calculated from each model. Undulations were computed from Topex altimetry and augmented with OSU91A to degree 360 when necessary.

Model	Mean difference (m)	rms difference (m)	Minimum (m)	Maximum (m)	Percent difference > 1 m
OSU91A	0.42	0.31	−2.81	5.07	1.1
GEM-T3	0.42	0.41	−4.27	5.31	3.7
JGM-1	0.43	0.28	−2.39	5.24	0.9
JGM-2	0.43	0.28	−2.48	5.24	0.9
TEG-2B	0.42	0.30	−2.50	5.22	1.0
GRIM4-C3	0.43	0.41	−6.99	4.48	2.1
WGS-84	0.41	0.79	−6.60	7.44	14.3

The most promising plan was the Aristoteles gradiometer mission proposed jointly by NASA and ESA and scheduled to fly in 1995; this project was recently terminated because of lack of funding. Currently, the most attractive of the proposed dedicated gravity satellite missions is the two orbiting satellite-to-satellite tracking Games mission proposed by GSFC. It will use laser transceivers and GPS transmissions for Doppler recovery from a passive satellite and is scheduled for launch in 1998. Table 8 briefly describes the objectives and status of recently proposed advanced gravity satellite missions, some with dual-use applications, that could be used for short-wavelength gravity model recovery and estimation.

Gravity Modeling Trends

More accurate Earth gravitational models continue to be developed. The current best civilian model is the joint model, JGM-2, which reflects a combination of data from NASA/GSFC, UT, OSU, and CNES. For the long-wavelengths regime, this model is complete to degree and order 70 based on satellite surface gravity and satellite altimetry measurement data; for the high-resolution short-wavelength region, the field is complete from degree and order 71 to 360 if the OSU91A model is used.

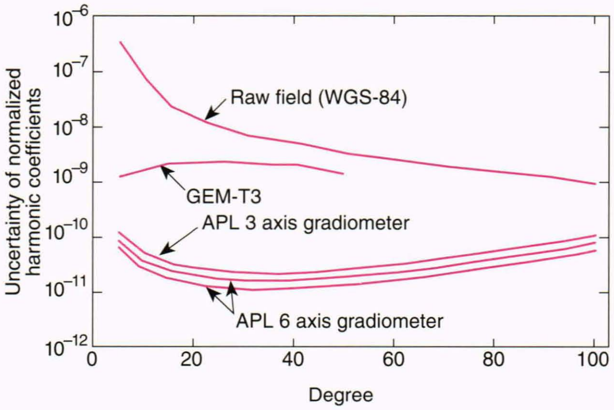

Figure 7. Uncertainty of normalized harmonic coefficients, showing the predicted increase in accuracy with the use of data from orbiting gravity gradiometers. Predictions for the proposed APL 3 and 6 axis gradiometers are based on 0.01 EU uncertainty for each in-line gradient measurement, observation of all gradients, a 4-s data interval for 7 months, and a circular polar orbit at an altitude of 200 nmi.

Future developments will focus on integrating data resources used for DoD's WGS-84 model and the joint JGM-2 model to construct a high-resolution collaborative model complete to degree and order 360. This high-resolution model will likely be made available to the geodetic and astrodynamics communities by mid-1996. The spherical harmonic coefficients describing both the new model and the geoid will be made freely available to the civilian community.

Most work on the new model will be done by NASA/GSFC with the same software used to construct the JGM-2 model; the updated gravity model will be computed using current satellite tracking data (e.g., Topex and GPS data), data from newly launched satellites, and new surface gravity data supplied by DMA's Aerospace Center. The DMA also will provide mean gravity anomalies from previously unavailable 30 × 30 arc-min global regions and additional altimeter data from the Geosat mission. The Naval Surface Warfare Center (Dahlgren Laboratories, Virginia) will provide Doppler tracking data from selected satellites not currently represented in JGM-2. The geoid resulting from the high-resolution surface gravity data will be used by DMA to update and improve the WGS-84 geoid.

SUMMARY

The attempt to standardize models of the Earth's gravitational field and shape began in 1961, when various investigators published gravitational constants in the form of low degree and order spherical harmonic coefficients based on Sputnik, Vanguard, Explorer, and Transit satellite tracking data. The major organizations associated with early gravity model analysis and development were SAO, NASA, APL, the U.S. Army Map Service, and The Royal Aircraft Establishment in the United Kingdom. In the late 1950s and early 1960s, standard gravity models used within the astrodynamics community for

Table 8. Proposed orbiting gravity missions.

Orbiting gravity mission	Concept	Mission objectives	Status
Games (NASA/CNES)	Satellite-to-satellite tracking (350–400 km polar orbit) • Cactus accelerometer (France) • Laser interferometer • Magnetometer (vector/scalar) • GPS receiver	Measure gravity field 100 times better than Topex to degree 80 and use 2 × 2 global block sizes (3-year lifetime) Orbit laser transceiver for Doppler recovery from a passive satellite	1998 launch date
Tides (NASA/GSFC)	Low–low satellite tracking (600-km polar orbit) • Laser interferometer	Map degradation in ocean current variability with geopotential signals	Concept definition phase
Aristoteles (NASA/ESA)	Two-dimensional gravity gradiometer in low polar orbit (300-km orbit) • GPS receiver • Gravity gradiometer • Two-axis accelerometer	Measure high-frequency gravity field to degree greater than 100 (6-month lifetime)	Canceled due to lack of funding
Stanford University Test of Equivalence Principle (STEP)	Very accurate supercooled quantum interference device (SQUID) accelerometer in orbit (1E−4 EU) (high eccentricity, 550 km) • GPS receiver	Test of equivalence principle Determination of *G* to 1 part per million Measure accuracy of low-order gravity field	Postponed to 2006
Gravity Probe-B (Stanford U/ NASA)	High-accuracy/precise gyro in orbit (600-km, high inclination) • GPS receiver • Retro-reflectors • High-quality gyro	Use general relativity to measure geodetic precession and drag of inertial reference frames (6-month lifetime)	Shuttle test in 1996

Projects Mercury, Gemini, Apollo, and other satellite programs consisted of a mix of gravity coefficient estimates with widely varying uncertainties. Sparse observations, inaccurate measurement systems, and the primitive computer programs used for processing observations prevented greater accuracy.

The first gravity models complete to degree and order 8 were published by APL in 1965 (APL3.5) based on the analysis of Transit Doppler tracking data and by SAO in 1966 (SAO 66) based purely on Baker–Nunn camera observations. Refinements in the models increased after NASA established the National Geodetic Satellite Program in the mid-1960s, with other organizations beginning to play a more active role in expanding the size and fidelity of the models available within the astrodynamics community in the United States and Europe.

Five gravitational models are currently in use within the scientific community. They were produced by a joint U.S.–European gravity team (JGM-2), DoD (WGS-84), UT (TEG-2B), OSU (OSU91A), and the European Community (GRIM4-C3). These models are now the best available, and they are equally applicable for astrodynamic applications requiring accuracies in the range of a few meters. However, for more precise applications where accuracies of a few centimeters are needed, the JGM-2 model is recommended: it is complete to degree and order 70 for describing the long wavelength of the gravity field and is supplemented with the OSU91A surface gravity extension to degree 360 when high spatial resolution is required.

Gravitational models will continue to evolve on the basis of additional observations and tracking systems. However, quantum leaps in satellite-based recovery—to degree and order 100 and higher—will require more sophisticated satellite-based instrumentation systems, such as can be provided by a combination of onboard gravity gradiometer, Doppler laser interferometer, and GPS measurements.

Gravity models will continue to be updated as processing methodology and models improve and as additional measurement data are incorporated into their architectures. Within the next year, the UT TEG-2B model will be replaced by the TEG-3 model, which will use Topex data. Sometime soon, NASA's JGM-2 and DoD's WGS-84 models will be combined to utilize the more extensive DoD-based surface gravity database available from the U.S. Naval Oceanographic Office. These are fairly minor refinements. They will improve fidelity in the high-order gravity model estimates and covariances to increase model accuracy for precision studies within the scientific and astrodynamics communities.

Further advances will occur largely through developments in theoretical modeling; improvement in the accuracy of satellite models; improved and expanded surface gravity measurements, particularly in unsurveyed areas of the world; improved ground- or orbit-based instrumentation; and the availability of supercomputers and workstations to enable easier and more efficient solution of large-scale computational problems.

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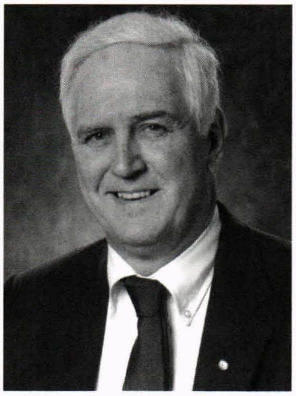

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