

PRECISION ACCELEROMETERS FOR GRAVITY GRADIENT MEASUREMENTS

An accelerometer, based on a microwave sapphire resonator, is under development for application in a compact gravity gradiometer. This gravity gradiometer potentially can be used in space gradiometry, precision navigation, and geodesy. The principle of operation of the accelerometer is based on measurements of classical spring-proof-mass displacements, which are converted to phase shifts in the output signal of a sapphire microwave resonator transducer. A prototype accelerometer has demonstrated a figure of merit sensitivity [$Q(df/dx)$] of 3.9×10^2 MHz/ μm . With a proof mass of 4500 g and a cantilevered beryllium-copper spring having a spring constant of about 60,000 N/m, this result corresponds to the detection of micro-*g* accelerations for low-frequency (<1 Hz) components of the acceleration vector by means of a precision frequency reference source.

INTRODUCTION

A gravity gradiometer is an instrument that measures the spatial rate of change of the components of the gravity field. In contrast to a gravimeter, which measures the strength of the gravity field, a gradiometer is more sensitive to the high-frequency spatial variations in the field. Variations in gravity are caused by variations in mass. Since gravity gradients decrease faster with distance than the gravity field itself, a gradiometer is sensitive to nearby changes in mass whereas gravimeters are strongly influenced by more remote perturbations. As a result, a gradiometer is a particularly useful tool for detecting buried objects and nearby geological voids. Gradiometers are also used to survey the high-frequency variations in the Earth's gravity field. Because a gradiometer measures force or acceleration gradients, it is not sensitive to the linear acceleration of the platform supporting it, rendering it particularly suitable for use on moving vehicles (including aircraft and spacecraft).

Many different types of gravity gradiometers have been proposed as instruments for measuring gravity gradients.¹⁻⁷ In general, gradiometers consist of pairs of accelerometers, incorporating proof masses and springs, that undergo differential acceleration when acted upon by the gravity gradient. The change in position of the proof mass in each accelerometer is linearly related to the amplitude of the acceleration. This displacement is detected by coupling the accelerometers to an electronic system that converts the displacement of the mass into an electrically measurable signal. The gradient is obtained from the difference between the accelerations measured by the accelerometers.

Most gravity gradiometers use some type of capacitive coupler, piezoelectric sensor, or magnetic field superconducting quantum interference device as detectors. Since the resolution of the gradiometer depends on the measurement sensitivity of the accelerometer, the design of the accelerometer is critical to the development of the

gradiometer. Gravity gradiometers that use superconducting quantum interference devices have been designed for resolutions of $<10^{-3}$ E · Hz^{-1/2} [1 Eötvös unit, E, is a change in gravitational acceleration of 10^{-9} m/s² (10^{-10} g or 0.1 μGal) over a baseline of 1 m].⁸ Thus, for resolutions smaller than 1 mE to be achieved, superconducting detection techniques are used to measure the extremely small displacements of the proof mass.

Gradiometers that use superconducting quantum interference devices, however, are too large to be transported easily. A transportable gravity gradiometer that uses a pendulous force rebalance accelerometer with capacitive pickup rings has been developed,⁹ but this device with its support equipment is the size of a large motor home. Furthermore, it has demonstrated resolutions of 10 E rather than the desired resolutions smaller than 1 E.

The design of the APL gradiometer described in detail in this article does not focus on improving the resolution of existing gradiometers but rather on producing a simpler and more compact instrument that will not require superconducting devices to detect proof mass displacements. This approach greatly simplifies the design of the gravity gradiometer as well as its operating logistics. The resolution of this prototype gradiometer is limited by its design to about 1 E, depending on its configuration. However, it will still be able to yield useful information for a variety of missions.^{10,11} With a sensitivity of 1 E, for example, the gradiometer could be used to detect underground voids 5 m in diameter at a depth of 25 m. If faster integration times (<1 s) were used, the gravity gradients associated with these subterranean voids could also be mapped at higher data rates, enabling the gradiometer to be placed on a moving platform.

The approach adopted at APL for high-resolution measurement of proof mass displacements involves using a microwave resonator transducer cavity as a displacement detector.¹¹⁻¹⁴ In this article we describe the design of such

an accelerometer in which the displacement of a proof mass, subject to the gravity force, modulates the resonant frequency of a sapphire microwave (9.6-GHz) resonator. By placing the sapphire resonator in a phase bridge, we can easily measure the phase modulation of a reference signal induced when the proof mass undergoes a displacement. We will present details about the prototype microwave transducer design and discuss data on initial accelerometer performance.

GRAVITY GRADIENTS

As a conservative force field, the gravity field can be determined from a potential function $\phi(x)$ at each point. The gravitational force is the field components and the derivatives of $\phi(x)$:

$$g(x) = \frac{\partial \phi}{\partial x}, \tag{1}$$

etc., using the geodesy sign conventions rather than the – (minus) sign in physics. The gradients of the gravity force, which are the rates of change of each field component in each direction, are thus the second derivatives of the potential and can be represented by a second-order tensor

$$\Gamma = \begin{bmatrix} \frac{\partial^2 \phi}{\partial x^2} & \frac{\partial^2 \phi}{\partial x \partial y} & \frac{\partial^2 \phi}{\partial x \partial z} \\ \frac{\partial^2 \phi}{\partial y \partial x} & \frac{\partial^2 \phi}{\partial y^2} & \frac{\partial^2 \phi}{\partial y \partial z} \\ \frac{\partial^2 \phi}{\partial z \partial x} & \frac{\partial^2 \phi}{\partial z \partial y} & \frac{\partial^2 \phi}{\partial z^2} \end{bmatrix}, \tag{2}$$

which is symmetric owing to the properties of the field. As a result, six of the nine terms are independent gradients. For locations where mass is not present (outside the solid Earth, for example), the gradients satisfy Laplace’s equation,

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0, \tag{3}$$

leaving only five independent components.

Those gravity gradients where the variation of the field component is taken in the same direction as the unit vector for the component are called “inline” gradients. Such a gradient can be measured by a pair of accelerometers whose sensitive axes are parallel and along the separation vector. In the following example the inline gradient of the x -component of the field in the x direction is being estimated. The outputs of two oppositely directed accelerometers along the x axis can be expanded by a Taylor series

$$\begin{aligned} g(1) &= g_x(0) + x \frac{\partial g_x}{\partial x} \\ g(2) &= -g_x(0) + x \frac{\partial g_x}{\partial x}, \end{aligned} \tag{4}$$

leading to

$$\frac{\partial g_x}{\partial x} = \frac{g(1) + g(2)}{2x}, \tag{5}$$

where the separation is equal to $2x$.

If five independent inline gradients are measured, all five independent gradient functions can be determined at the same point. The cross gradients, such as $\partial^2 \phi / \partial x \partial y$, are related by rotations to inline gradients that are not along the coordinate axes. There are, however, many practical advantages to having a symmetric array of sensors. Thus, the proposed system will consist of six inline accelerometer pairs oriented along the diagonals of a regular dodecahedron (Fig. 1). In this configuration the trace of the gradient tensor, which is known to be zero, will also be measured. This measurement will help to ascertain the level of noise of the gradiometer.

ACCELEROMETER FOR GRAVITY GRADIOMETRY

The proposed gravity gradiometer consists of six pairs of accelerometers placed on a symmetric dodecahedral mount (Figs. 1 and 2). The distance between opposite accelerometers is about 30 cm, and the proof masses weigh about 1 kg each. The electronic and mechanical systems, consisting of the microwave phase detector and thermal and vacuum systems, are placed beneath the actual gradiometer. On the basis of the present design of the mass spring system, the sensor part of the gradiometer

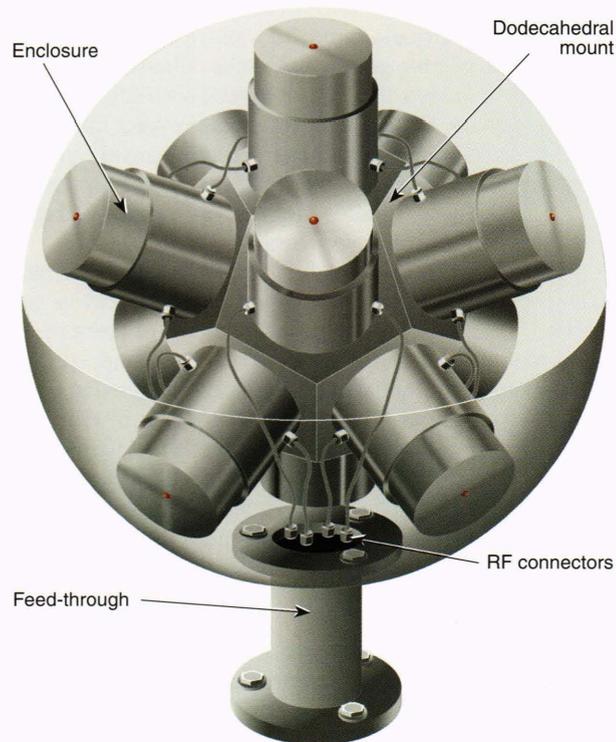


Figure 1. Gravity gradiometer configuration showing six pairs of colinear accelerometers placed on a dodecahedral mount.

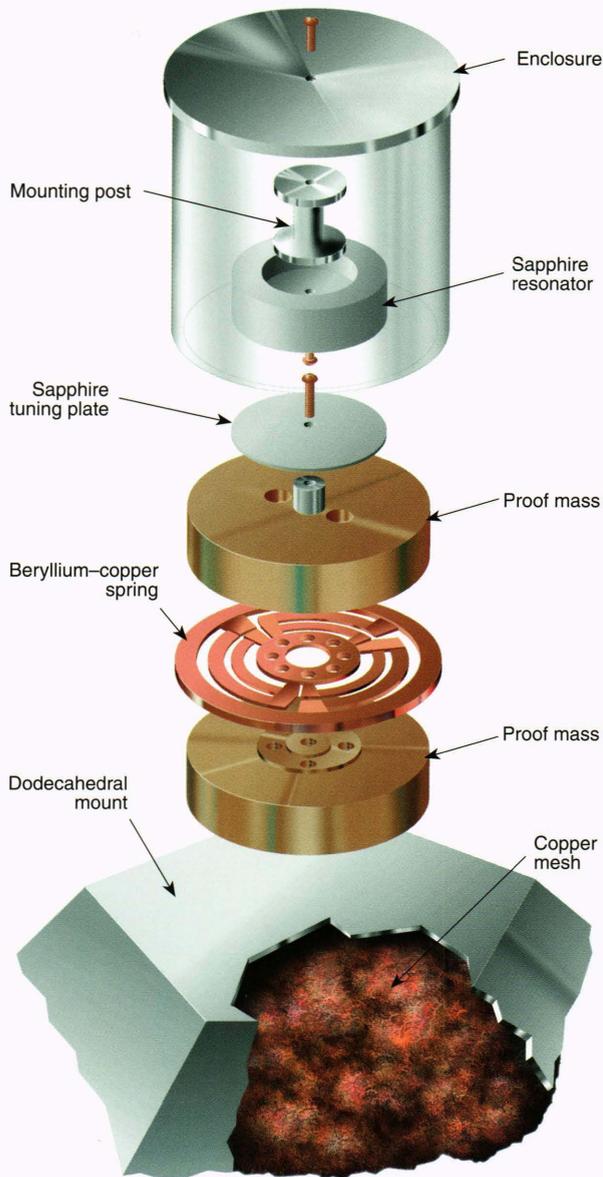


Figure 2. A simplified diagram of a single accelerometer.

is about 75 cm in diameter and weighs about 25 kg. Its support equipment takes up about one standard-sized equipment rack.

In order to assess the proposed technique, we developed a set of accelerometers for evaluation of mechanical characteristics and microwave resonator sensitivity. Each accelerometer consists of a set of proof masses suspended symmetrically on a beryllium-copper spring. This spring has a diameter of 4 in. and a spring constant of approximately 60,000 N/m. The design is a derivative of springs originally developed for Stanford University's Gravity Probe B Program.¹⁴ The mechanical characteristics (linearity, spring constant, and quality factor) of the spring mass system were investigated by mechanical resonance techniques. For this purpose, proof-mass displacements were measured with a microwave cavity transducer as

well as with a laser interferometer.^{10,15} The spring in the APL accelerometer has a mechanical resonance frequency of about 17 Hz with a single 4.5-kg proof mass attached to the hub of the spring (Figs. 3a and 4a and b).

We investigated the operation of the microwave transducer by modifying a sapphire resonator, originally developed as a microwave signal source, and integrating it with the spring mass system (Figs. 3a and b). This microwave resonator is designed in a whispering gallery mode and is made of single crystalline sapphire with the optic or c axis perpendicular to the plane of the resonator. This type of resonator supports high quality factor (Q) microwave whispering modes, which have been the topic

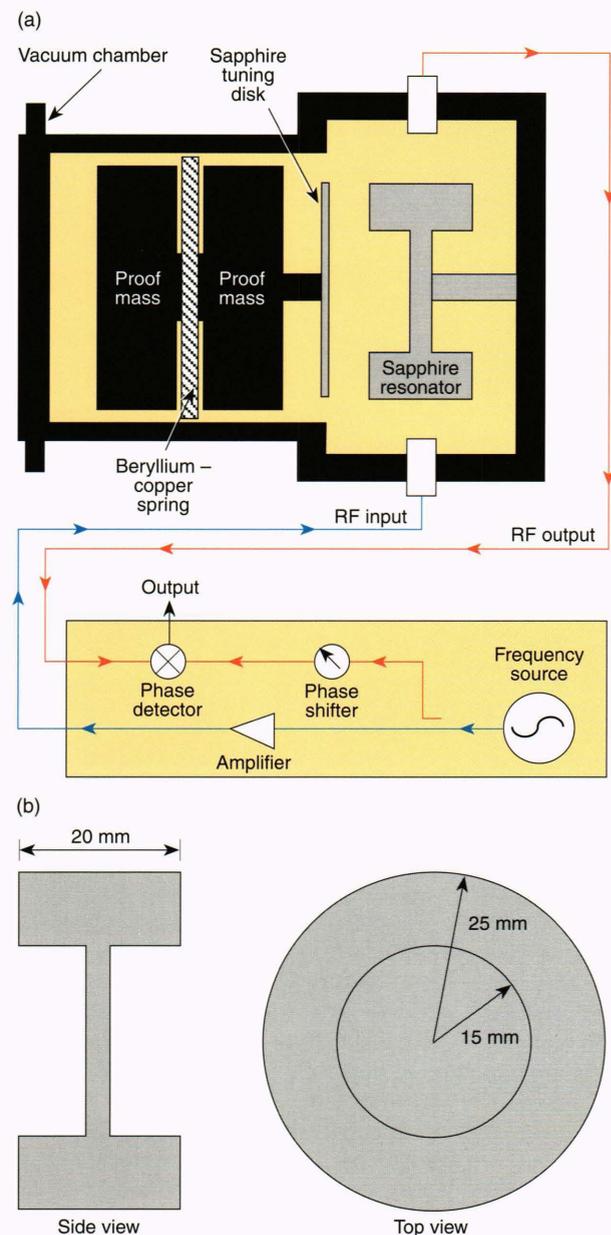


Figure 3. Details of an accelerometer: (a) the spring and proof mass displacement phase detector circuit and (b) a cross section of the sapphire resonator.

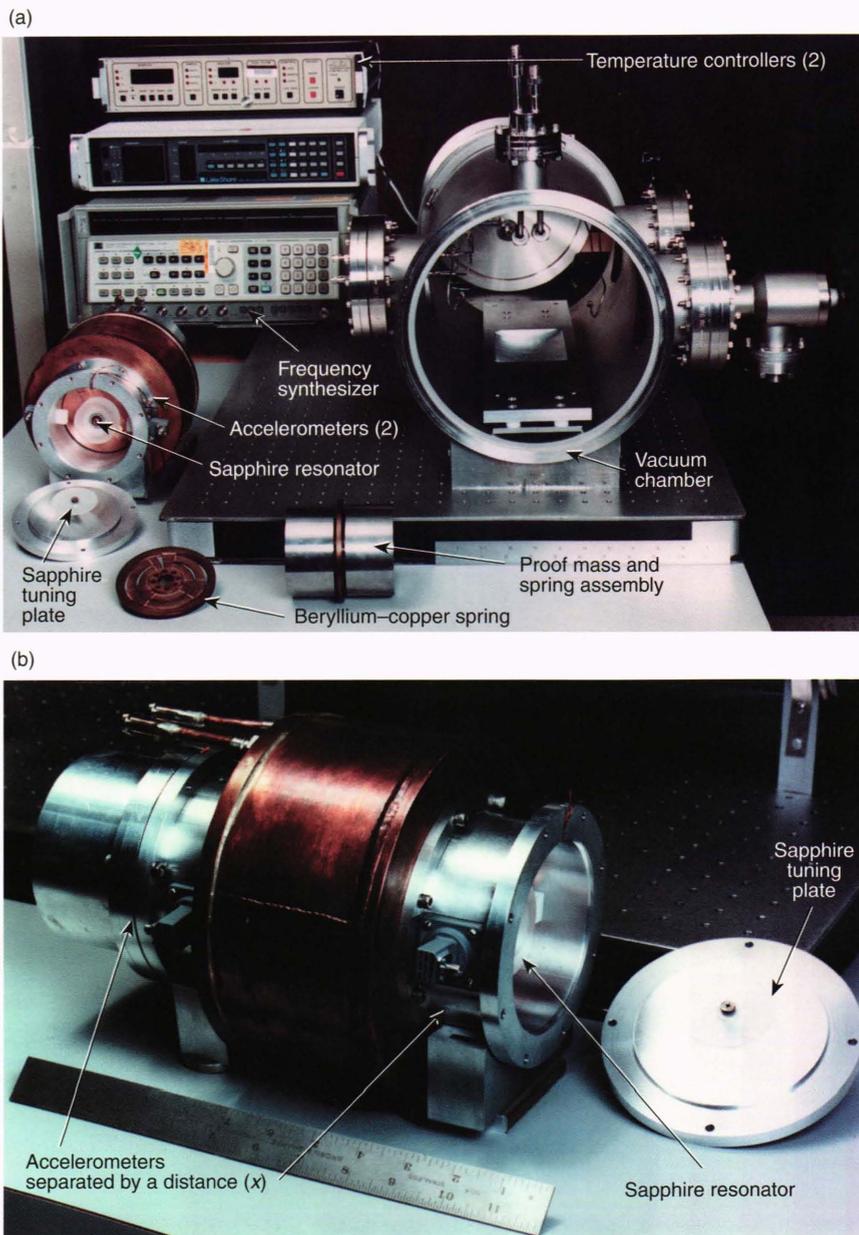


Figure 4. Photographs of the prototype one-axis gradiometer showing (a) the laboratory test set-up, including the sapphire cavity resonator, spring, and proof masses, and (b) the accelerometer assembly. Measurements reported in this article were made with this prototype.

of many investigations.^{13,16,17} The name “whispering mode” is derived from the acoustic analogy of an audio resonance that can be observed in the rotunda of St. Paul’s Cathedral in London. There, acoustic signals are reflected against the walls of the rotunda and can be heard by an observer anywhere along those walls. The microwave version works the same way: electromagnetic waves are internally reflected inside the sapphire ring resonator. Experiments showed that the sapphire resonator operates in a hybrid whispering gallery mode¹¹ with a resonant frequency of 9.6 GHz, having a loaded Q of 73,000.

As shown in Fig. 3b, the sapphire resonator has the configuration of a rectangular ring supported by a thin inner webbing. The outer and inner radii are 25 and 15 mm, respectively, and the ring’s thickness is about 20 mm. A sapphire tuning disk was attached to one of the masses and placed at a nominal distance of 2.5 mm from

the sapphire microwave resonator. The resonator is coupled to a reference signal from a frequency synthesizer that has an output power of -5.0 dBm. A change in position of the sapphire disk with respect to the sapphire resonator introduces a small change in the resonance frequency of the resonator. This phase shift is detected with a DC coupled mixer yielding a voltage (V_ϕ) measured as a function of displacement of the tuning disk.

A least-squares analysis of the data in Fig. 5 yields a tuning constant of $dV_\phi/dx = -0.371$ mV \cdot μm^{-1} . The tuning constant dV_ϕ/dx may also be expressed¹¹ in terms of the characteristic phase detector constant K_ϕ and the slope of the phase change $d\phi$ against displacement dx

$$\frac{dV_\phi}{dx} = K_\phi \frac{d\phi}{dx}, \tag{6}$$

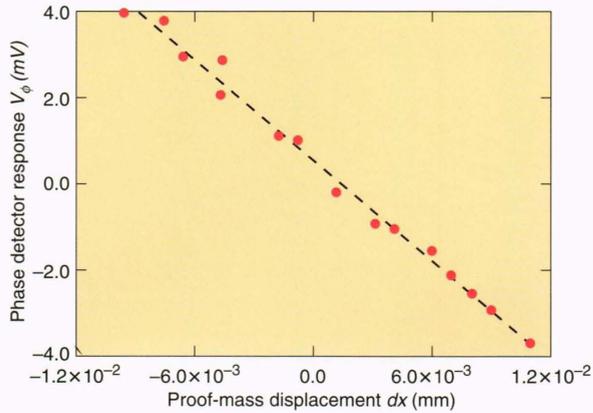


Figure 5. Phase detector signal as a function of proof-mass position. The measurements were taken at a frequency of 9.6 GHz and at room temperature. The slope is $dV_\phi/dx = -0.371 \text{ mV} \cdot \mu\text{m}^{-1}$.

where

$$\frac{d\phi}{dx} = \frac{2Q}{f_0} \frac{df}{dx}. \quad (7)$$

Given a maximum value of dV_ϕ/dx at a resonance $f_0 = 9.6$ GHz, the figure of merit $Q(df/dx)$ for the present accelerometer equals $3.9 \times 10^2 \text{ MHz} \cdot \mu\text{m}^{-1}$. Given this value for $Q(df/dx)$, displacements of about 10^{-8} m could be resolved with a microwave transducer using a reference source with a frequency stability (dff/f_0) equal to 10^{-10} for sampling frequencies smaller than 1 Hz. This result implies that micro-*g* accelerations can be resolved with the accelerometer if the sapphire resonator transducer is adequately stabilized.¹¹

These initial results have demonstrated the proof of principle for using sapphire resonator transducers as displacement detectors in accelerometers. Additional work will be needed to increase the transducers' sensitivity and frequency stability. For gravity gradiometry, the relationship between the gravity gradient (Δg) and the differential proof mass displacement (Δx) is⁴

$$\Delta x = \frac{x(\Delta g)}{(\omega_m)^2}, \quad (8)$$

where x is the baseline between the two accelerometers (30 cm) and ω_m represents the mechanical resonance frequency of the beryllium-copper spring [$(2\pi)17$ rad/s]. Hence, a gravity gradient (Δg) of 1 E produces extremely small differential displacements (Δx) of about 2.8×10^{-13} m. Further improvements to the accelerometer are necessary before this instrument would be able to detect gradients of this magnitude.

Some of these improvements are currently being addressed by increasing the phase detector's tuning constant K_ϕ . By reducing the spacing between the sapphire tuning disk and the resonator, the parameter df/dx (Eq. 7) will also be increased, improving the sensitivity of the transducer and spring-mass accelerometer. A series of design modifications has been implemented that addresses the

mechanical properties of the spring as well. A new spring has been designed that has a spring constant of about 17,000 N/m and a mechanical resonance of 30 Hz with a proof mass of 1 kg. Its diameter has also been reduced by a factor of 2, which will further reduce the size of the accelerometers and enable the baseline (Δx in Eq. 8) to be shortened.

CONCLUSION

Using a classical spring-proof-mass system coupled to a sapphire microwave (9.6-GHz) resonator transducer, we have demonstrated a sensitivity, $Q(df/dx)$, of $3.9 \times 10^2 \text{ MHz} \cdot \mu\text{m}$. With this sensitivity the beryllium-copper spring and proof-mass accelerometer can resolve accelerations at the micro-*g* level. To increase the resolution of the accelerometer, we are redesigning the spring-mass system by reducing the spring's size so that its spring constant will be reduced to about 18,000 N/m. Future improvements will address the electromagnetic coupling between the sapphire tuning disk and the resonator. Sensitivity can also be increased by reducing the spacing between the tuning disk and resonator; optimizing the phase detector circuit's tuning factor (K_ϕ); replacing the tuning disk with a second resonator, similar in design to the microwave resonant cavity, for increased electromagnetic coupling; and improving the mechanical rigidity and temperature stability of the accelerometer.

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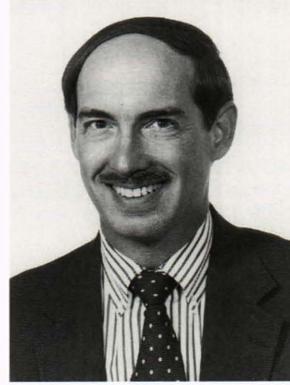
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THE AUTHORS



JOSEPH J. SUTER received a B.S. degree in physics and mathematics from the Free University of Amsterdam, The Netherlands, in 1977. He received an M.S. degree in physics from Michigan State University in 1980 and an M.S.E.E. degree from the University of Maryland in 1983. In 1988, he was awarded a Ph.D. degree in materials science and engineering from The Johns Hopkins University. Dr. Suter joined APL in 1983 and is a Principal Professional Staff scientist and Supervisor of the Space Department's Time and Frequency Section. He is a member of the IEEE, APS, SPIE, and Sigma Xi. He has served on several IEEE committees on time and frequency technology. Dr. Suter was appointed a research associate in the Department of Materials Science and Engineering of The Johns Hopkins University in 1993.



PAUL A. ZUCKER received a B.S. degree in physics from the University of Chicago in 1966 and an M.S. degree from Stanford University in 1967. He was awarded a Ph.D. degree in physics from Stanford University in 1971. Dr. Zucker joined APL in 1975 after postdoctoral appointments at the Universities of Minnesota and Oregon. At APL he developed an error model simulation for inertial submarine navigation systems and has been continually involved with the evaluation of ballistic missile accuracy models. He currently is a physicist in the Strategic Systems Department, where he has been active in the evaluation and validation of gravity error models. This activity led to the study of passive navigation and gravity gradiometry. Dr. Zucker is a member of the American Physical Society, AAAS, American Geophysical Union, Sigma Xi, and Phi Beta Kappa.



L. PETER MARTIN received a B.S. degree in 1987 and an M.S. degree in 1993, both from The Johns Hopkins University Department of Materials Science and Engineering. Since 1991 he has been a graduate research assistant and Ph.D. candidate in the Department of Materials Science and Engineering. In 1993, Mr. Martin passed his graduate board examinations and received an APL fellowship for his work on gravity gradiometers. He is currently in the final stages of his doctoral degree requirements.