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DELTA-K RADAR MEASUREMENTS OF INTERNAL WAVES IN THE SOGNEFJORD

A multifrequency radar of the type known as a delta-k radar makes it possible to probe the ocean surface roughness directly at scales much longer than the wavelength of the radar. The application of a delta-k radar to the measurement of ship-generated internal waves in the 1988 Sognefjord experiment is described. The results of this experiment agree with predictions for the delta-k response and demonstrate the potential utility of delta-k radar as a new tool in radar oceanography.

INTRODUCTION

Remote sensing of the ocean surface by satellite or airborne radars can be accomplished in several ways.[1,2] An imaging radar, such as a synthetic aperture radar, can produce a high-resolution image of the ocean surface where each point in the image represents the radar reflectivity of a small area on the ocean surface. Because the degree of surface reflectivity is a function of the roughness of the ocean, and because the roughness is a result of the combined action of the local winds, waves, and currents, the radar image can be used to infer something about the characteristics of these winds, waves, and currents. Synthetic aperture radar oceanography was the subject of a previous issue of the *Johns Hopkins APL Technical Digest*.[3]

Another type of radar useful in remote sensing of the ocean surface is a scatterometer, which illuminates a large area of the ocean surface and measures the average reflectivity but does not form an image. Scatterometers are used to measure wind speed and direction on the basis of both the strength of the reflected radar signal and its dependence on the pointing direction of the radar.[1]

Satellite altimeters are a third type of radar useful in oceanography. An altimeter measures the distance from the satellite to the ocean surface to determine the shape of the surface. This shape is a function of ocean currents as well as of spatial variations in the Earth's gravitational field. If the gravitational field is known, the currents can be inferred. Satellite altimetry is treated elsewhere in this issue of the *Johns Hopkins APL Technical Digest* and was also discussed in a previous issue.[4]

Still another type of radar, one that has not yet become a conventional part of ocean remote sensing, is a multifrequency radar commonly called a delta-k radar. The name is derived from the fact that the wavelength and propagation direction of the energy radiated by a radar are denoted by its wave vector, **k**. The delta-k radar uses two or more different wavelengths (and possibly illumination directions) and processes the radar return in a way that achieves some of the effects of a radar that transmits energy corresponding to the difference of two **k**-vectors. A discussion of the delta-k radar and its application to the detection of internal waves and the measurement of their frequencies is developed here. In particular, we discuss some of the delta-k results obtained from a joint United States–Norway experiment conducted in the Sognefjord, Norway, in July 1988.

DELTA-K RADAR PRINCIPLES

Delta-k radars have been used for several years[5-7] and can be operated in either a continuous-wave or pulsed mode. The differences between the two modes are important in some applications, but they do not affect the application discussed; the continuous-wave mode will be discussed because it is conceptually simpler.

The delta-k radar can be thought of as two separate radars transmitting on closely spaced frequencies. For example, two of the frequencies used in the Sognefjord experiment are 6.8422 and 6.8402 GHz. The wavelength for these frequencies is about 0.044 m. The backscatter received by the radar at each frequency is called the single-frequency response. Because the two frequencies are close together, the backscatter at each will be very similar; that is, we expect the same backscattered power and the same shape for the Doppler spectrum.

The delta-k signal is formed by multiplying the received signal at one frequency by the received signal at the second frequency and processing the product to retain only the low-frequency components (components near the difference frequency). If the two frequencies are equal, the result is a signal with zero phase and an amplitude that will vary as the strength of the radar backscattered power varies; but if the two frequencies are unequal, the result is a signal with a phase that changes with time. The frequency of the delta-k signal—the rate at which the phase changes—will tell something about the nature of the ocean surface.

The delta-k concept is illustrated in Figure 1. Figure 1A shows two modulated sinusoids in which the frequency difference of the two is equal to the modulation frequency. The frequency of each sinusoid represents the transmitted frequency. The modulation represents the spatial variations in the reflectivity of the ocean surface.

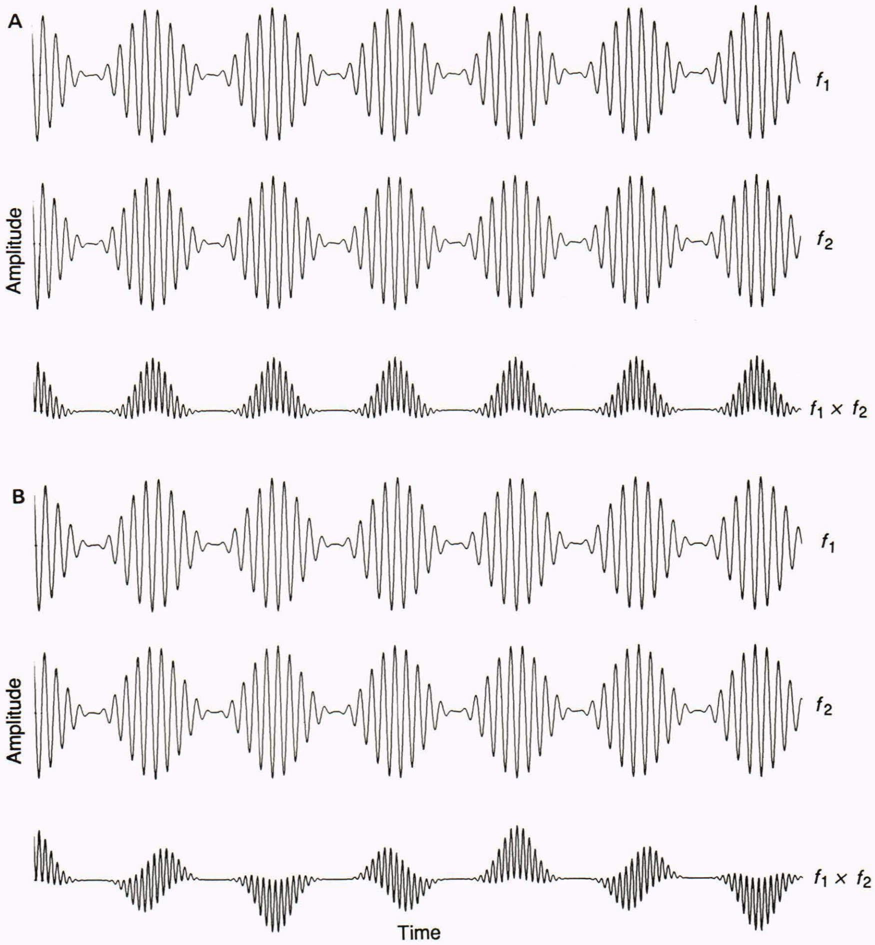

Figure 1. Conceptual operation of a delta-k radar. A. Difference frequency = modulation frequency. B. Difference frequency = 0.75 modulation frequency.

The delta-k processing begins by multiplying these two received signals. The product signal is positive everywhere. When the product signal is averaged, it will indicate the presence of a delta-k signal. In Figure 1B, the modulation frequency is the same as before, but the difference frequency is reduced by 25%. The product signal is equally positive and negative and averages to zero. No delta-k signal results when the modulation frequency and the difference frequency are unequal.

If the surface of the ocean is moving with a velocity v toward a stationary radar, the single-frequency backscatter will then be shifted in frequency (relative to the transmitted frequency) by

$$f_D = \frac{2vf_1}{c} , \tag{1}$$

where f_1 is the transmitted frequency and c is the speed of light. This is the normal Doppler shift. A 1-m/s velocity in the look direction of the radar will produce a Doppler shift of 46 Hz for the frequencies just mentioned.

The Doppler frequency of the delta-k signal is given by

$$f_D = \frac{2v(f_1 - f_2)}{c} , \tag{2}$$

where f_2 is the second frequency. The shift is identical to that for a single-frequency radar, except that the frequency is now replaced by a frequency difference. (It has been assumed here that the two frequencies have been transmitted and received from the same point in space and that the radar is stationary. If the radar is moving, v would be the relative velocity between the radar and the surface.) For a 2-MHz frequency difference, for example, the effective wavelength of the delta-k radar is 150 m, which is the wavelength of a 2-MHz radar. A 1-m/s velocity in the look direction of the radar will produce a delta-k Doppler shift of only 0.013 Hz. Observation of this small Doppler shift requires integration for several minutes, in contrast with the single-frequency Doppler shift, which can be determined accurately with data gathered over only a few seconds.

Because the ocean surface does not simply move as a whole toward the radar but involves a very complicated motion, a delta-k signal actually consists of a range of frequencies instead of a single Doppler frequency. Understanding the shape of this frequency spectrum and its dependence on oceanographic and radar parameters is the goal of ongoing research.

In addition to the large difference in the magnitude of the Doppler shift produced by single-frequency and delta-k radars, these radars differ in the amount of ocean surface they can illuminate. If we are interested in the reflectivity of the ocean at wavelengths of a few centimeters, an area of just a few square meters needs to be illuminated; but if we are interested in the delta-k properties of the ocean surface, an area of a few hundred square meters must be illuminated when the difference frequency is 2 MHz. Illuminating such a large area is not a problem for ocean remote sensing, but it does mean that research on delta-k radar performance must be done at greater distances from the ocean surface than research on single-frequency radars.

An important variation of the delta-k radar concept involves the use of two or more radars located in different positions, in which case the difference wave vector can be oriented in some direction other than the one to which the individual radars are pointed. For example, with this technique the radar could be "tuned" to detect waves moving in the cross-range direction instead of only detecting waves moving in the range direction.[5]

DELTA-K RADAR RESPONSE TO INTERNAL WAVES

The response of a delta-k radar to ocean surface roughness patterns can be predicted from any given model of the scattering of electromagnetic waves by a rough surface. As part of the work described here, a model of the response of a delta-k radar to a statistically described ocean surface has been developed on the basis of the Kirchhoff approximation of the scattering. Another assumption built into the model is that the surface of the ocean is a Gaussian process—that the random height of the ocean surface at one position and time and the random height at another position and time obey a bivariate Gaussian distribution. The result of these approximations and assumptions is a somewhat complex model of the delta-k radar response. For the most part, though, the strength of the delta-k radar response is proportional to the modulation strength of the radar reflectivity of the ocean surface at length scales that are half the delta-k wavelength. This is the Bragg condition. (Remember, we use delta-k wavelength here for the wavelength of radiation that has a frequency equal to the delta-k difference frequency.)

Associated with the delta-k radar is a Bragg wave number, k_B, which is twice the horizontal component of the difference wave vector for the two frequencies used to create the delta-k signal. That is,

$$k_B = \frac{4\pi(f_1 - f_2)}{c} \sin\phi ,$$

where ϕ is the incidence angle of the radar.

Although internal waves do not directly displace the ocean surface to an appreciable extent, their propagation results in surface currents interacting with the short waves that resonate with the radar energy to produce the backscatter. A periodic internal wave produces periodic variations in the amplitude of these short waves, as shown in Figure 2. If a very large area of the ocean is illuminated by the radar, a delta-k response will be observed only when the wave number of these internal waves, k_{IW}, equals the Bragg wave number of the radar. In this simple portrayal, the response of the delta-k radar to each pair of transmitted frequencies tells us about the strength of internal waves that have a wavelength matching the Bragg wavelength of the radar.

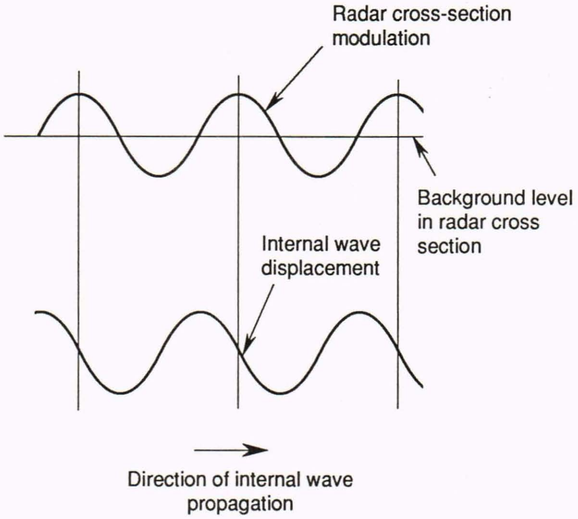

Figure 2. A periodic internal wave (bottom) produces periodic variations in the amplitude of the radar cross-section modulation (top).

In the situation described, the delta-k radar responds only to waves that are propagating directly toward or away from it and that have a wavelength equaling the Bragg wavelength of the radar. In practice, the radar will respond to waves with other wavelengths and to waves that propagate at angles other than toward or away from the radar. The extent of the radar response to these other internal waves is determined largely by the dimensions of the ocean surface area illuminated by the radar. As the range dimension of the illuminated area decreases, the radar response to waves that do not have the Bragg wavelength increases. As the azimuth dimension of the illuminated area decreases, the radar response to waves propagating in directions other than toward or away from the radar increases. (Range is the direction from the radar to the segment of ocean area being illuminated. Azimuth is the direction perpendicular to range.) Illumination of a very small area results in a radar that responds to internal waves of all lengths and directions, making it a poor sensor for these waves.

To quantify this radar response, assume that the radar illumination pattern on the ocean surface is Gaussian. The power incident on the surface will fall to $1/e$ of the power in the center of the area at a distance X in azimuth and a distance Y in range. Refer to Figure 3 and note that the angle between the direction of propagation of the internal waves and the direction to the radar line of sight is θ. Where x and y represent the position of the area increment relative to the center of the illuminated area, the radar response to waves propagating in this direction with wave number k_{IW} will be proportional to

$$\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\, e^{-ik_B y}\, e^{-(x^2/X^2 + y^2/Y^2)} \times \cos[k_{IW}(x \sin\theta + y \cos\theta)]$$

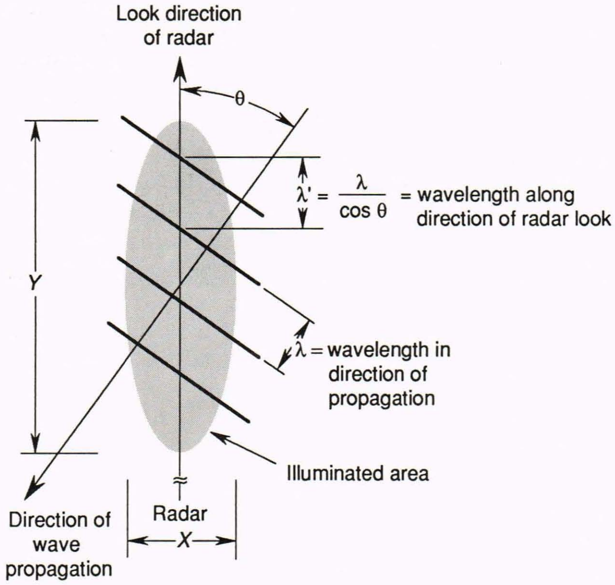

Figure 3. If the radar illumination pattern on the ocean surface is Gaussian, the power incident on the surface will fall to 1/*e* of the power at the center of the area at a distance *X* in azimuth and a distance *Y* in range. Note that *θ* is the angle between the propagation direction of the internal waves and the look direction of the radar. The wavelengths along the two directions are related by λ = λ′cos *θ*.

$$
\begin{aligned}
&= \frac{\pi XY}{2}\exp(-\tfrac{1}{4}k_{\mathrm{IW}}^2 X^2 \sin^2\theta) \\
&\times \{\exp[-\tfrac{1}{4}Y^2(k_{\mathrm{IW}}\cos\theta - k_{\mathrm{B}})^2] \\
&+ \exp[-\tfrac{1}{4}Y^2(k_{\mathrm{IW}}\cos\theta + k_{\mathrm{B}})^2]\} \ .
\end{aligned}
$$

The integral does not take into account how much wave energy is actually present in a given wave component. It only represents the relative response of the delta-k radar to that energy. For a given wave number k_{IW} and angle θ, the Bragg wave number k_{B} that will produce the maximum response is

$$k_{\mathrm{B}} = \pm k_{\mathrm{IW}}\cos\theta \ .$$

The waves can be traveling either toward or away from the radar, but the response will be weak unless

$$|k_{\mathrm{IW}} X \sin\theta| \lesssim 2 \ .$$

The result shows how the delta-k radar response depends on both the direction and the wavelength of internal waves. So far we have considered the direction and wavelength to be independently variable, but if we observe ship-generated internal waves, we see that the direction and angle of the internal waves are coupled together. Knowledge of this fact is useful in predicting the response of a delta-k radar to such waves.

Like waves traveling in any medium, internal waves are characterized by a dispersion relation that defines the frequency of oscillation as a function of the wave vector. For light in a vacuum, the frequency is proportional to the wave number. For waves longer than about 10 cm on the ocean surface, the frequency is proportional to the square root of the wave number. For internal waves, the frequency has a dependence on the wave number that is determined by how the water density changes with depth. Given the profile of the water density, the dispersion relation of the internal waves can be computed.

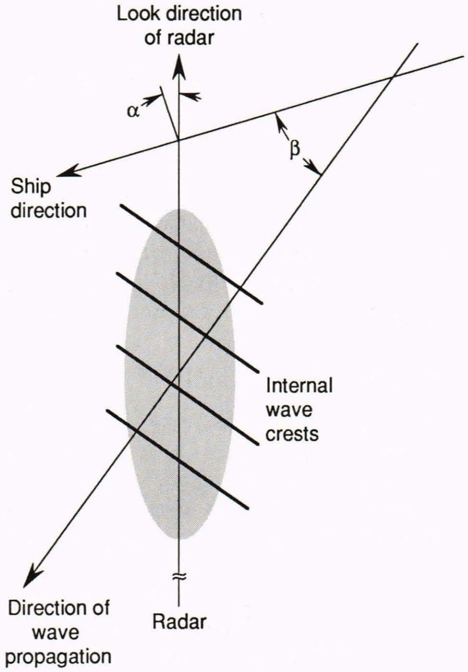

Figure 4. Geometrical relationship of ship, radar, and waves.

A ship moving at a constant velocity generates waves moving in different directions and having different wavelengths, but these waves must meet the condition that

$$v k_{\mathrm{IW}} \cos\beta = \omega(k_{\mathrm{IW}}) \ ,$$

where v is the ship speed, k_{IW} is the wave number of the ship-generated wave, β is the angle between the ship motion and the wave propagation, and $\omega(k_{\mathrm{IW}})$ is the frequency of the wave. This condition is equivalent to the statement that the ship-generated waves are stationary in the reference frame of the ship (a property of waves generated by a ship moving at a constant velocity).[8] It should be remembered that the angle of propagation, β, is a function of the wave number, k_{IW}.

The angle by which the radar look direction differs from perpendicularity to the ship track is designated α, as shown in Figure 4. The angles α and β in Figure 4 are related to the angle θ in Figure 3 as follows:

$$\theta = \frac{\pi}{2} - \alpha - \beta \ .$$

For a given value of k_{IW}, we can compute β from a knowledge of the ship speed, v, the angle of the ship track, α, and the dispersion relation, $\omega(k_{\mathrm{IW}})$. The Bragg wave number of the radar that will produce the greatest response to these waves is

$$k_B = \pm k_{IW} \cos\theta = \pm k_{IW} \sin(\alpha + \beta) .$$

The strength of the response at this Bragg wave number is proportional to

$$\exp[-\tfrac{1}{4} k_{IW}^2 X^2 \cos^2(\alpha + \beta)] .$$

The Doppler shift in the radar backscatter produced by the internal waves is simply the frequency of these waves, $\omega(k_{IW})$. We have here the necessary tools to predict the delta-k radar response to ship-generated internal waves.

THE SOGNEFJORD EXPERIMENT

The Sognefjord experiment is described in the article by Apel and Gjessing, elsewhere in this issue. The several features that make the Sognefjord an especially useful site for a delta-k experiment are the steep hillside, the power cables across the fjord, and the presence of highly stratified water.

The steep hillside provides a long viewing time from a stable position at moderate incidence angles. (The angle of incidence is conventionally defined relative to the normal to the surface.) The importance of this combination is that a quantitative test of the models of the delta-k radar can be made at incidence angles for which we have confidence in our understanding of the radar backscatter. Most delta-k measurements of the ocean surface have been made at or near grazing incidence (viewing angles nearly parallel to the water surface) by using radars on towers[6] or cliffs.[7] Some measurements at smaller incidence angles have been reported from aircraft, but those measurements do not allow for the long integration times necessary for achieving the resolution required by delta-k processing. At large incidence angles, the radar backscatter problem is further complicated because some parts of the ocean surface are shadowed by waves and because horizontal advection of short surface waves by longer waves can affect the shape of the Doppler spectrum of the backscatter. The Sognefjord experiment was an opportunity to test the delta-k theory in a regime where such complications are not present.

Two radars were positioned on the hillside overlooking the fjord. One was a 6.8-GHz radar operated by the Royal Norwegian Council for Scientific and Industrial Research; it is a six-frequency, vertically polarized, continuous-wave radar. The other radar was a 10-GHz radar operated by Metratek, Inc., of Falls Church, Va.; it is a five-frequency, horizontally polarized, pulsed radar.

The electrical power cables across the fjord provided a unique opportunity to make delta-k radar measurements of internal waves from incidence angles as small as 10°. Land-based radars cannot make this kind of measurement, because even if a sheer cliff were found at the water's edge, the waves would be too near the shore to have the properties of waves in open water. A tower cannot be used, because the radar would be too near the surface to illuminate a sufficiently large area with a nearly constant incidence angle. A fixed-wing aircraft would not allow sufficient integration time. Helicopters, airships, or aerostats are possible alternatives and do have several advantages, but even these do not have the stability of a radar suspended from a cable. During the experiment, two radars were carried by a gondola that traveled along the power cables. (The gondola is normally used for maintenance by the Norwegian Power Board.) The radars were a six-frequency, 4.5-GHz radar and a single-frequency, 96-GHz radar, both operated by the Royal Norwegian Council for Scientific and Industrial Research.

In addition to the hillside and the presence of the power cables, the Sognefjord contains a combination of seawater and fresh water that makes it easy for surface ships to generate internal waves. Because of the combination of factors, the Sognefjord was well suited for a delta-k internal wave experiment.

THE SOGNEFJORD RESULTS

The results obtained from the 4.5-GHz radar at an incidence angle of 40° during runs 3 and 4 on 12 July 1988 follow, and Table 1 lists the relevant parameters for the data. The radar transmits and receives six frequencies, each individually offset from the reference signal by an amount ranging from 50 to 79 MHz. The individual offsets and the possible difference frequencies afforded by them are shown in Table 2. The frequency differences range from 2 to 29 MHz; the corresponding Bragg wavelengths range from 117 to 8 m.

The single-frequency Doppler spectrum-versus-time plot generated for this radar is shown in Figure 5 for the time interval corresponding to runs 3 and 4. The frequency of the Bragg waves at the center of the illuminated area is about 5.7 Hz. The energy is spread over a range of frequencies and shifted up in frequency by the action of those surface waves that are much longer than the Bragg waves. A 10-Hz Doppler shift for this geometry corresponds to a horizontal velocity of about 0.51 m/s. The spectra exhibit considerable variability, but the lack of distinctive features indicates that no internal waves are present.

The predicted delta-k response to ship-generated internal waves is shown in Figure 6. The predictions are based on a calculation of the dispersion relation for internal waves that can be generated by the ship when the ship speed is 2 m/s and the angle between the radar look direc-

Table 1. Parameters for runs 3 and 4 on 12 July 1988.

Parameter	Value
Incidence angle, ϕ	40°
Azimuth extent, X	79 m
Range extent, Y	105 m
Ship speed, v	2 m/s
Ship angle, α	20°

Table 2. Frequencies and frequency differences.

Channel numbers and frequency offsets (MHz)							
1 (50)	2 (52)	3 (57)	4 (67)	5 (76)	6 (79)	Δf (MHz)	Bragg wavelength (m)
x	x					2	116.7
				x	x	3	77.8
	x	x				5	46.7
x		x				7	33.3
			x	x		9	25.9
		x	x			10	23.3
			x		x	12	19.4
	x		x			15	15.6
x			x			17	13.7
		x		x		19	12.3
		x			x	22	10.6
	x			x		24	9.7
x				x		26	9.0
	x				x	27	8.6
x					x	29	8.0

Note: The x's occur in pairs and mark the frequency offsets that result in the difference frequency Δf.

tion and the direction of ship travel is 20°. The dispersion relation is computed from *in situ* measurements of the water-density profile made during the experiment. The possible difference frequencies are indicated by the upper scale on the plot. The delta-k response is predicted to fall to about half of its maximum value at a difference frequency between 7 and 9 MHz. The Doppler shifts will be less than 0.02 Hz.

Figure 7 shows the delta-k Doppler spectrum versus time at a frequency difference of 5 MHz. The plot contains three bands, each about 2 minutes long, corresponding to discontinuities in the radar data. The duration of the bands is 2 minutes because that is the time span of each Fourier transform. Two features in Figure 7 correspond to the two ship runs on 12 July. Unlike the spectrum shown in Figure 5, the delta-k spectrum shows well-defined features that are manifestations of the internal wave passage.

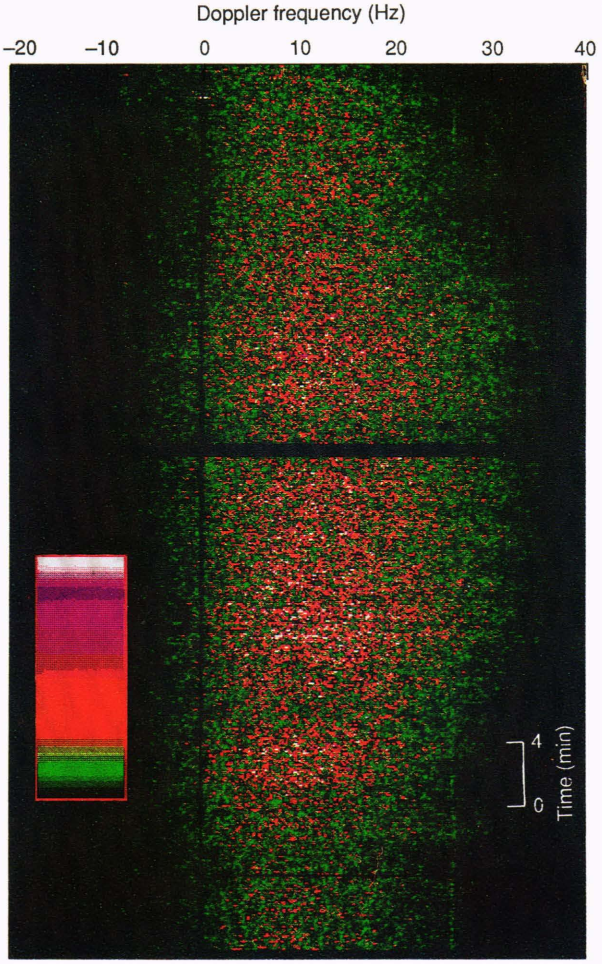

Figure 5. Single-frequency Doppler spectrum versus time for the time corresponding to runs 3 and 4. The frequency of the Bragg waves at the center of the illuminated area is about 5.7 Hz.

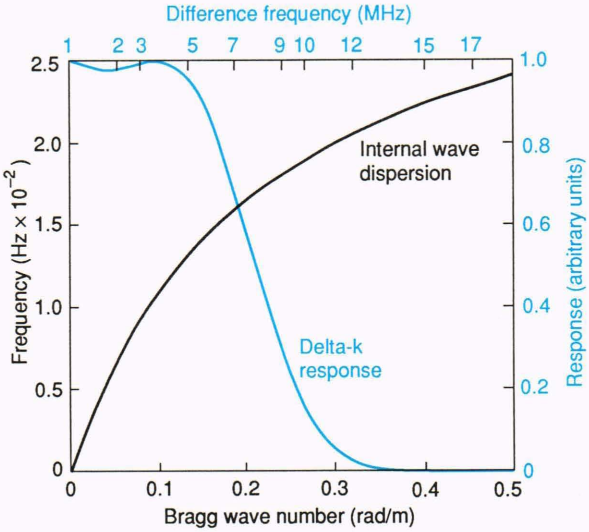

Figure 6. Predicted response of delta-k radar to ship-generated waves.

Figure 8 shows the Doppler spectrum obtained by taking a cut through this plot (Fig. 7) at a time when the

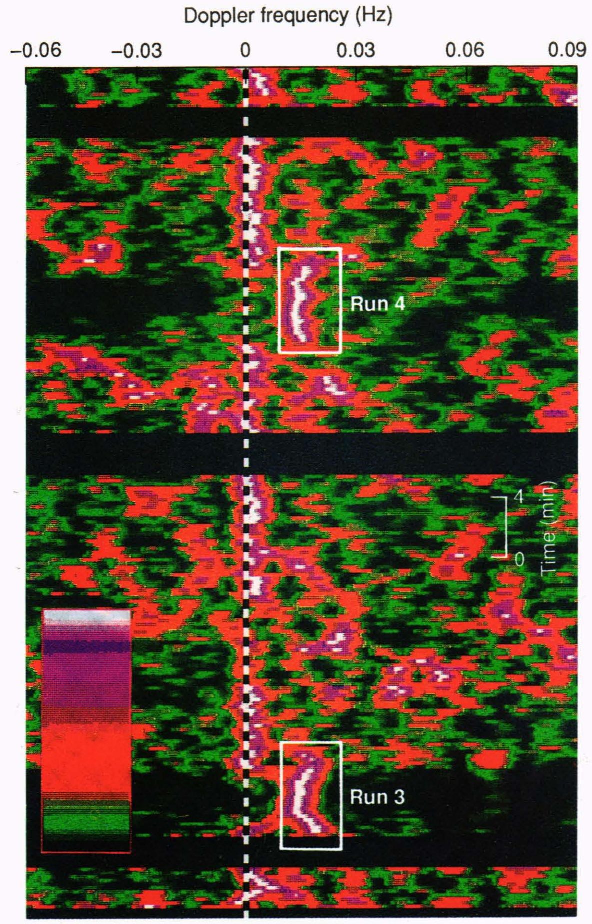

Figure 7. Delta-k spectrum versus time for a difference frequency of 5 MHz for runs 3 and 4.

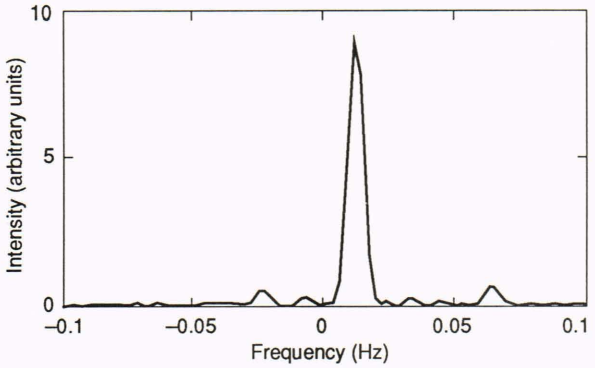

Figure 8. A cut through the delta-k spectrum during run 3 on 12 July 1988.

internal wave signal from run 3 is present. The peak in the spectrum is at 0.016 Hz, which compares favorably with the predicted internal wave dispersion curve for the fjord stratification on 12 July.

The same procedure was followed for other frequency differences of 7, 9, 10, and 12 MHz to find the positions of the peaks in the corresponding Doppler spectra for both run 3 and run 4. In some cases, a clear delta-k peak was not observed. Figure 9 shows the positions of the peaks in the cases where a peak was observed. Very good agreement was achieved between the Doppler shifts observed with the delta-k radar and the predicted internal wave dispersion relation.

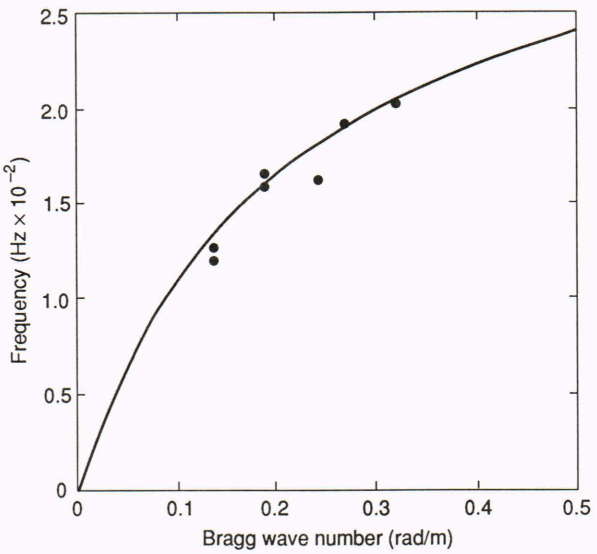

Figure 9. Doppler shifts plotted against the predicted dispersion curve of Figure 6.

An additional investigation was made of the timing of the arrival of the internal waves. The peaks in the delta-k spectra appear at a later time for the larger frequency differences than for the smaller ones, consistent with the greater group speed for long waves (which resonate with small frequency differences) compared with that for short waves. The waves all travel the same distance from the track of the wave-producing ship to the area illuminated by the radar; therefore, the faster-traveling waves appear first. The time of ship passage, the distance traveled, and the predicted group speeds were used to predict the arrival time of waves in the radar footprint. The predicted times and the observed times agreed very well. From these results, we concluded that the delta-k radar is a potential tool for measuring the propagation of internal waves where *in situ* measurements are not available.

In addition to the 4.5-GHz radar discussed, the Sognefjord experiment produced delta-k results for the 6.8-GHz hillside-mounted Norwegian radar and the 10-GHz pulsed Metratek radar. Analyses similar to those discussed here have also been done for those radars. The various radars were found to have different sensitivities and to perform best under different conditions. Current research is oriented toward obtaining a better understanding of the optimal match between the environmental conditions, the internal wave properties, and the radar design parameters. The results obtained have clearly demonstrated that delta-k radars are useful for detecting internal waves at moderate incidence angles and for measuring their frequencies. Such radars are expected to find a place as another standard tool in radar oceanography, alongside imaging radars, scatterometers, and altimeters.

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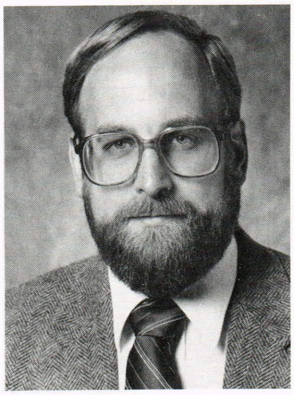

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