LAWRENCE E. PAYNE

MEMBRANES, PLATES, WAVEGUIDES, AND WATER SLOSHING

ESTIMATING EIGENVALUES WITH A POSTERIORI/A PRIORI INEQUALITIES
James R. Kuttler and Vincent G. Sigillito, *The Johns Hopkins University Applied Physics Laboratory*
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In this monograph, the authors present a new method for estimating eigenvalues and eigenvectors of a real symmetric operator A (usually positive definite and unbounded), a method that couples a general a posteriori inequality with an operator-specific a priori inequality. If the appropriate explicit a priori inequality is known (and this is the case for most problems of physical interest), the method gives a simple, easily implemented procedure for obtaining simultaneous upper and lower bounds for the eigenvalues of A. The method is developed in detail for numerous membrane, plate, waveguide, and water-sloshing problems and is used to compute close upper and lower bounds for several of the smaller eigenvalues in each case.

The authors' method has certain advantages over previous approaches. First, the approximating functions need not belong to the domain of the operator A. For example, in the clamped membrane problem, the approximating functions do not have to satisfy the homogeneous boundary conditions, as is required in most other methods. A second advantage is that upper and lower bounds for the eigenvalues are obtained simultaneously, whereas most other methods compute upper and lower bounds using different techniques and approximating functions. In fact, both upper and lower bounds are obtained using a single linear combination of simple functions, and, if several eigenvalues in the same problem are to be computed, many of the computations can be made once, stored, and used in computing each of the eigenvalues.

The one somewhat unsatisfactory feature of their method is the requirement of additional a priori information in order to determine which eigenvalue (eigenfunction) is being approximated. There is usually little difficulty in identifying the first few eigenvalues in a given symmetry class, provided these eigenvalues are sufficiently separated. Of course, few practical approximation methods yield close bounds for higher eigenvalues, and most methods require special care when two or more eigenvalues are close together.

The monograph is self-contained and written in an easily readable style, with simple examples used for motivation. The material is readily understood by the engineer who might not be well versed in functional analysis. Numerous examples illustrate the practicality of the method introduced in the monograph, and, for the convenience of the user, the authors have given a complete FORTRAN listing of all problem-independent subroutines needed for implementing their method.

The need for sharp bounds for eigenvalues arises in many contexts, but such bounds are of particular importance today in the study of various types of nonlinear phenomena. In many important physical problems that are modeled by initial and/or boundary value problems for nonlinear partial differential equations, close bounds for one or more of the smaller eigenvalues of a related linear problem are often required in order to derive sharp explicit criteria for existence, uniqueness, regions of asymptotic stability, decay rates of solutions, and the like. In the reviewer's opinion, the method presented gives a good (perhaps the best) method for computing the sharp bounds that are needed.

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