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INTERNAL SOLITARY WAVES IN THE SULU SEA

Large internal oscillations of a density-stratified body of water often occur in the form of nonlinear solitary waves, or solitons. Recent field and theoretical investigations have delineated much of the dynamics of these waves, including the processes of generation, propagation, and attenuation. Some groups of solitons measured in the Sulu Sea near the Philippines have amplitudes of up to 100 meters, wavelengths in excess of 10 kilometers, and lifetimes of 2½ days. Their major properties may be reproduced theoretically in a satisfactory way.

INTERNAL WAVES

Gravity waves on the surface of the sea are perhaps the most visible example of a class of fluid motion falling under the rubric of geophysical fluid dynamics. Ocean surface waves have long occupied the attention of intellectuals ranging from natural philosophers through Freudian psychologists to poets. A less-well-appreciated (and less-visible) type of motion called an internal gravity wave also exists and has been known to oceanographers for over 75 years. Such waves may occur within the body of a fluid whose density varies in the vertical, whenever that fluid is disturbed by some means. The vertical density gradient may come about through temperature variations (largely resulting from solar heating of the upper layers of the ocean), salinity variations (usually resulting from lighter, fresher water from rivers or fjords flowing across colder, saltier water below), or, least important, fluid compressibility (resulting from the weight of the overlying water).

Internal gravity waves are most familiar in devices sold in novelty stores wherein two immiscible fluids differing only slightly in density are confined to layers in a closed, clear plastic box. By quickly tilting the box, one may excite a large, slow, low-frequency wave propagating along the fluid interface, which then reflects off the ends to oscillate back and forth as a kind of internal seiche. The amplitude of the wave, which is zero at the top and bottom of the box because of the rigid boundaries there, has its maximum at the interface between the fluids. This internal wave is of low frequency because of the small density change, $\Delta\rho$, across the interface; the buoyancy provided to an upwardly displaced parcel of water by the surrounding lighter fluid reduces the downward acceleration of gravity, g, to an amount equal to the actual gravity times the fractional density change. This quantity, $g\Delta\rho/\rho$, is termed the effective gravity, g_{eff}, and in many geophysical fluids is as small as 1/1000th of the actual value of g. The radian frequency, ω, of an internal oscillation is thereby decreased by the square root of this ratio, and the period of an internal wave is correspondingly much longer than its surface wave equivalent, all else being equal.

Internal waves also travel more slowly than surface waves. To appreciate this, recall that a small-amplitude, long surface wave in a shallow channel filled with fluid to a depth, h, will travel with a phase speed $c = (gh)^{1/2}$. If such a channel is instead filled to the same depth with two layers of fluid that differ slightly in density, as above, with the two layer depths being h_1 and h_2, an internal wave excited on the interface between the layers will now move with a speed $c_0 = (g_{eff}h_{eff})^{1/2}$, where the effective gravity is as defined above, but where the effective depth, h_{eff}, is the harmonic mean of the layer depths: $h_{eff} = h_1 h_2/(h_1 + h_2)$. This effective depth is always smaller than the depths of either the upper or the lower layer. Thus, with both a reduced gravity and a small effective depth, internal waves propagate very slowly indeed. Owen M. Phillips discusses both surface and internal waves in his excellent book.[1]

A surface gravity wave mode still exists in the two-fluid channel, of course, as long as the upper surface is free, and it has essentially the same properties as in the single-layer case. In the parlance of geophysical fluid dynamics, the surface wave is known as the barotropic mode, while the internal wave is known as the baroclinic mode, the latter because surfaces of constant pressure (baro) are inclined with respect to surfaces of constant density in such modes.

Strictly speaking, an internal wave in the two-fluid box is an interfacial wave, but analogous internal waves exist in the continuously stratified fluids of the earth's atmosphere and oceans, and indeed on other planets having fluid envelopes. The combination of solar heating and gravity results in a stratification of a planetary fluid such that its density generally decreases with height. The atmosphere is usually most stable at night, when solar heating of the surface and the resultant unstable upward convection of air have ceased and the atmosphere has settled down to gravitational equilibrium. On the other hand, the ocean is most stable during the daytime when solar heating of the upper layers has rendered them lighter and hence more buoyant than the deeper waters. Under stably stratified conditions, any forces that excite vertical excursions of either of these fluids will generally result in the propagation of internal waves away from the source. Such vertical forces may arise from the flow of currents over topographic features (e.g., mountain ranges, the edge of the continental shelf, underwater sills, or sea mounts) and result in significant vertical motion of fluid at depth. They may also occur in currents where the flow velocity varies sharply with height; such shear flow can become unstable if intense enough and result in vertical excursions of the fluid that take on wavelike characteristics. Indeed, any forces that move the stratified fluid vertically may be regarded as candidate sources for internal waves.

If the stratified fluid is disturbed by some means, the disturbance propagates away from the point of origin as a train of waves characterized by polarization largely perpendicular to the direction of propagation and by a well-defined relationship among wavelengths, frequencies, and angles of propagation. That is, the waves obey a dispersion relation that relates ω to the wave vector, $\mathbf{k}$, via an equation of the form $\omega = \omega(\mathbf{k})$, which also implicitly involves the vertical density gradient and the water depth in addition to the wave number and direction. For small-amplitude linear waves, the dispersion relation does not depend on the wave amplitude, η, just as the frequency of a clock pendulum does not depend on the amplitude of its swing, to a first approximation. However, if the wave amplitude is large, either surface or internal waves may become nonlinear and the propagation characteristics thereby become dependent on amplitude in addition to the other dependencies mentioned.

Although a wide variety of nonlinear waves is known, a particular type called the solitary wave has come under very active investigation recently and considerable understanding of its unusual properties has been gained. In nature, solitary waves cover an enormous span of space scales, ranging from the hypothesized but as-yet-undiscovered magnetic monopole, whose theoretical dimensions are of order 10^{-12} centimeters, to the great red spot of Jupiter (thought to be a mode-2 solitary planetary wave), whose size of approximately 10,000 kilometers exceeds the diameter of the earth. The plasma physics version of the solitary wave is termed a soliton, although this terminology has been extended through less-than-careful use to include most other solitary waves as well.

PHYSICS OF SOLITARY INTERNAL WAVES

Strictly speaking, a solitary wave in or on a fluid is an isolated, propagating disturbance that moves over large distances without change of shape. Regarded as something of a hydrodynamic curiosity until relatively recently, the existence of a surface soliton was first noted during the last century by John Scott Russell,[2] who observed the generation of a "mound of water" caused by a tow barge brought to an abrupt stop in a shallow canal in England. The bow wave launched by the halted barge was described by Russell as an isolated pulse of elevation that propagated for several miles along the canal as he followed it on horseback. Struck by its solitary character and its relatively small change of shape, he dubbed the phenomenon a "solitary wave" and subsequently carried out controlled experiments to determine its properties. The essential element necessary to support this surface wave was found to be shallow water, with η required to be comparable with h in order for the solitary character to be established and maintained. Under a balance of body forces that comes into play with these conditions, as is explained below, the wave relatively quickly develops a characteristic shape and thereafter maintains its form indefinitely in the absence of spreading or dissipation. Subsequently, Korteweg and deVries[3] developed a theoretical explanation of the solitary wave phenomenon.

Twentieth century workers in hydrodynamics came to the realization that a stratified fluid could equally well support solitary internal waves but of a somewhat more varied and complicated form than the surface solitons of the English barge canal. It turned out that in a density-stratified fluid, the depth need not be shallow to support solitary waves—indeed the fluid could be arbitrarily deep—as long as the vertical scale of the depth region containing the significant density gradient was small enough.[4] The gradient-containing region acts as a kind of waveguide, confining the major portion of the wave energy to a relatively narrow vertical range while allowing propagation in the horizontal, and thus rendering the total water depth of lesser importance. If the wave amplitude is comparable to the scale of the gradient and also is steep enough, it may then move as a solitary wave; on the other hand, if the wave height is small, it will still propagate, but now as a linear wave.

Given that the amplitude of the internal disturbance is large enough for it to obey nonlinear dynamics, it will evolve from its initial form (which is mostly established by flow, stratification, and topography) toward a series of more or less isolated soliton pulses whose shapes are nearly independent of their origins. This process is illustrated schematically in Fig. 1, which shows a sequence wherein a strong tidal ebb over a steep underwater sill first generates an internal hydraulic jump downstream of the barrier. Upon tide reversal, this depression moves out to the right and develops undulations that eventually evolve

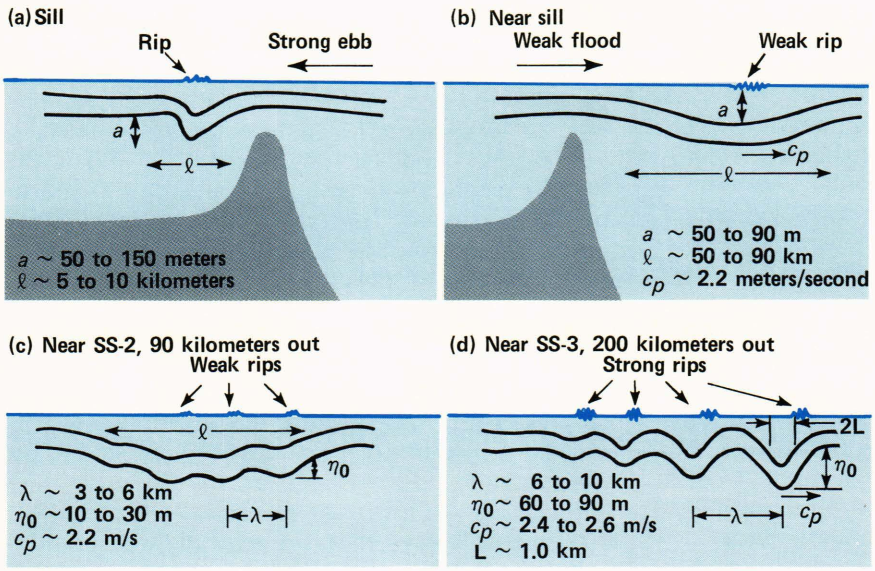


Figure 1 — Schematic showing processes responsible for the generation of a packet of solitons. (a) A strong ebb tide over a sharp underwater sill develops an internal hydraulic jump, or bore, downstream. A surface tide rip of amplitude *a* and length *l* overlies the depression. (b) Upon tide reversal, the depression slips over the sill and out into open water. (c) At 90 kilometers out, the bore has developed undulations and weak, rough striations on the surface. (d) At 200 kilometers out, large rank-ordered solitons have fully developed, speed has increased with amplitude, and considerable surface roughness accompanies each soliton. Further propagation leads to increased wavelength and diminution of amplitude, the latter resulting from radial spreading.

into a packet of rank-ordered solitons of large amplitude; these continue to draw apart as they propagate as a radiation field. Thus, the energy for the solitons comes from the tide, which is, in turn, extracted from the earth-moon-sun system.

The evolution is not difficult to understand because it results from two simple properties of waves in a bounded material medium that lead to a balance: the opposing forces of linear internal wave dispersion and nonlinear wave cohesion, or steepening. Linear dispersion is the characteristic possessed by waves wherein the Fourier components of the waves having different frequencies and wavelengths propagate at different speeds. In hydrodynamics, longer gravity waves travel faster than shorter ones and hence water waves generally are dispersive. A pulse consisting of many Fourier components tends to spread out in time and to disperse itself accordingly. The nonlinear cohesion, a kind of shock phenomenon, requires some restriction on propagation speed for its existence, such as that provided by limited channel or layer depth, as was discussed above. In such a layered medium, the finite wave amplitude, η, in essence deepens the layer depth as the wave goes by, so that the new effective depth becomes $h_{\mathrm{eff}} + \eta$, and the long-wave speed becomes

$$c = [g_{eff}(h_{eff}+\eta)]^{1/2} , \qquad (1)$$

which is greater than the propagation speed for small amplitudes. Thus, larger Fourier components tend to travel faster than small ones of the same wavelength and move toward the front of a complex wave pulse. A pulse consisting of such waves becomes narrower and steeper, and tends toward a shock condition. In the solitary wave, these opposing tendencies toward dispersion on one hand and shock formation on the other come into balance in the long run in a way that results in a wave of definite, fixed shape at large distances and times from the source. This is the solitary internal wave. Theoretically, it travels at higher-than-linear speeds and exists by itself, maintaining its shape and amplitude indefinitely. Furthermore, in deep water, the internal solitary wave is one of depression only, because of the buoyancy effects and the presence of a surface boundary.

In addition to their proclivity towards traveling without change of shape and towards propagating faster the larger they are, solitons possess other unusual properties, including a theoretical mutual phase retardation upon intersecting another soliton, and the tendency to fission into a number of smaller, shorter solitons under certain conditions of forcing. These and other characteristics are discussed in more detail in the references, to which the reader is directed for a deeper understanding of the phenomenon.[5]

While the maximum vertical displacement of a solitary internal wave occurs at depth, this does not imply that there are no surface manifestations. A free surface boundary will exhibit at least three surface modifications. The first is a barotropic, out-of-phase surface soliton analogous to the surface gravity wave mentioned above, whose amplitude is smaller than the internal displacement by a factor of $(\Delta\rho/\rho)$, or of about 0.1% of the maximum excursion at depth under typical oceanic conditions. The second and third surface manifestations both result from variations in the local surface wave amplitude due to straining by the horizontal currents of the solitary internal wave. The current strain rate, or horizontal derivative, tends to focus the overlying surface wave energy into a narrow region of rough water overlying the downgoing phase of the soliton, leaving the surface immediately behind the roughened region depleted of wave energy and hence relatively smooth. This leads to a modulation in surface wave energy that undergoes a transition from a normal sea state to a narrow, rough region to a somewhat wider, smoother region, and finally to normal roughness toward the rear of the soliton. This may be seen in Fig. 2, which shows a vertical profile of an internal soliton of depression propagating toward the right, accompanied by alternating rough and smooth regions. This process may be described formally in terms of surface-wave/internal-wave interaction theory, but its magnitude will clearly depend on surface-wave and internal-wave amplitudes, among other things.

The third modulation mechanism results from surface oils and surfactants that are swept up by the soliton and apparently carried along with it in the phase region of reduced roughness; the quantitative nature of this interaction is poorly understood. The resultant rough and smooth regions have fixed phases relative to the soliton that depend weakly on soliton amplitude, and they frequently propagate along with it and serve to delineate it on the surface.

SOLITONS IN THE SEA

In light of the variety and complexity of fluid motions that occur in the sea, it would be surprising if anything remotely resembling an internal soliton were observable in nature. Nevertheless, such is the case; the existence and properties of these waves have been firmly established by several investigators who have used various measurement techniques that range from bathythermograph temperature profiles to satellite imagery. Some of these investigations have been combined field and theoretical efforts that have contributed in a major way to the understanding of the waves, including work by Farmer and Smith,[6] Haury *et al.*,[7] Hughes,[8] Gargett,[9] Osborne and Burch,[10] and Apel and Gonzalez.[11] While the field investigations have concentrated on a few locations known to be sources of solitary waves (chief among them being the oceans off the United States northeast and northwest coasts), it has been high-resolution satellite imagery that has revealed the ubiquitousness of the phenomenon. LANDSAT and SEASAT, especially, have yielded a large number of images at sub-100-meter resolution showing solitons at a variety of sites around the world. Indeed the satellite data suggest that on a global scale, the continental shelf edges, island arcs, sea mounts, and bathymetric features that protrude into the upper 1000 or so meters of water are generally sources of internal waves wherever the tide is strong enough. Thus, statistically a global source function is implied.

The mechanisms responsible for the ability of overhead sensors to image events occurring beneath the sea surface are those mentioned above that modulate surface roughness. The roughness variations then differentially scatter incident sunlight (in the case of imagers operating at visible wavelengths, such as LANDSAT) or backscattered microwave power (in the case of imaging radars like the one on SEASAT). They thus serve to delineate the phase fronts

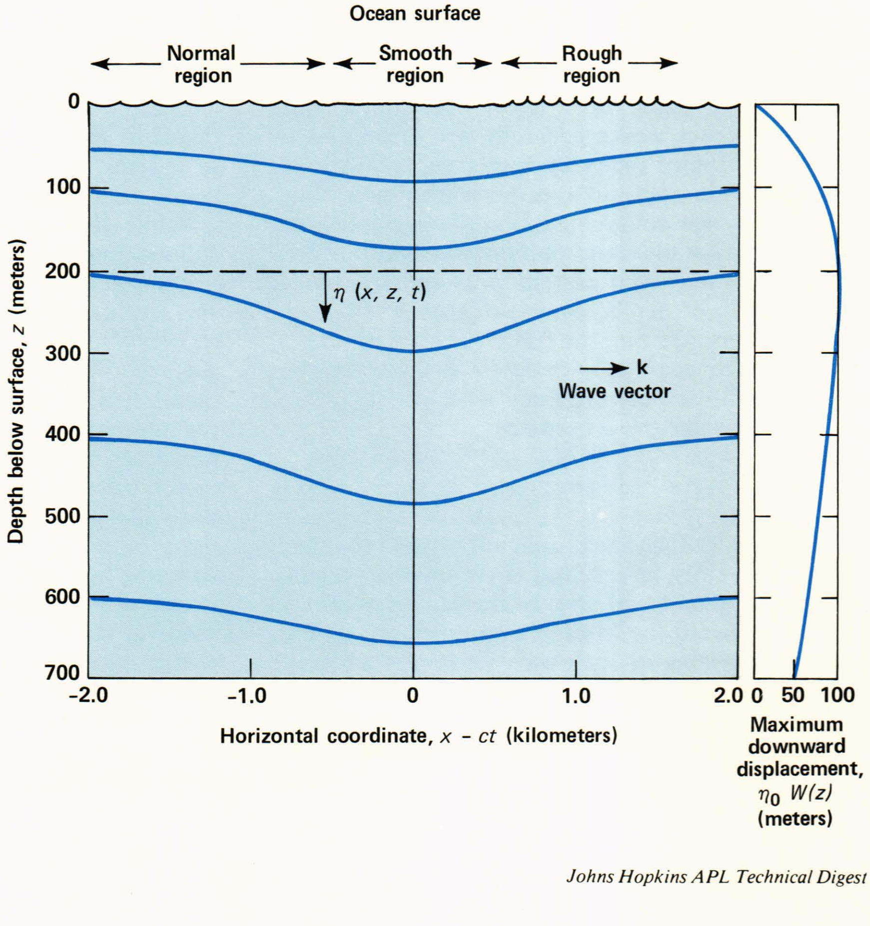


Figure 2 — A vertical (x,z) section of a single internal solitary wave of depression. Lines shown are excursions of constant density surfaces. The scale parameters are typical of those in the Sulu Sea near the Philippines, with a maximum amplitude, η_0, of 100 meters and a characteristic width, L, of 1 to 2 kilometers. Also shown schematically are regions of enhanced roughness and smoothness that delineate the position of the soliton on the surface.

of relatively coherent internal waves via broad-scale "snapshots" of the roughness modulation, under proper conditions. The identification of the striations in the imagery as internal solitons further depends on somewhat indirect interpretations of the imagery in terms of the nonlinear dynamics of tidally generated wave groups, and thus is model-dependent. Furthermore, it is only the very coherent, narrowband, larger amplitude internal waves whose surface signatures are recognizable in the images. Generally invisible to remote sensors is a large class of less coherent, smaller amplitude internal waves that are characterized by power-low spectra and small correlation lengths and that constitute the great bulk of open-ocean internal wave energy. While the in-water measurements are much more direct and less dependent on dynamical inference, they suffer from being confined to specific sites for limited periods and, hence, a lack of a synoptic, continuing overview.

For observations of coherent solitary internal waves, the combination of in-water observations with overhead sensing and theoretical interpretation constitutes a powerful technique that yields information not available from any one method alone. Such a program has been conducted by Apel *et al.*[12] and is described in what follows.

THE SULU SEA INTERNAL SOLITON EXPERIMENT

Very-large-scale internal waves have been observed at a variety of locations around the island archipelagoes of the Far East, with reports of what are almost certainly surface manifestations of such waves extending back at least a hundred years. Wallace[13] writes of several occurrences during the 1800's of narrow lines of propagating white caps passing sailing ships, or appearing as breaking waves on otherwise undisturbed beaches in the region. A century later, medium-resolution satellite images taken by the Defense Meteorological Satellite Program (DMSP) were interpreted by U.S. Navy personnel as showing very long (several kilometer) internal waves in the Sulu and East China Seas.[14] The Sulu Sea, which is an especially active region for internal waves, is an interior mediterranean sea bounded by the southwestern Philippines and Borneo. The DMSP images from there showed surface striations occurring in several packets, each of which contained a few to as many as 10 individual waves whose spatial and velocity scales were two to five times larger than those characterizing internal waves on the continental shelf off the U.S. east coast. Figure 3a is a DMSP image from April 1979 showing five separate packets of solitons as striations apparently radiating from a small source in the southern part of the Sea; Fig. 3b is a line drawing interpretation of the striations. Assuming tidal generation every 12.5 hours, the interpacket separation implies a propagation speed of about 2.4 meters per second; the wavelengths vary from about 5 kilometers near the source to greater than 16 kilometers on the far side of the Sulu Sea, near Palawan Island.

> Starting early, on the 4th of October, the same S.S.W. wind continued, and we began to fear that we should hardly clear the southern point of Gilolo. The night of the 5th was squally, with thunder, but after midnight it got tolerably fair, and we were going along with a light wind and looking out for the coast of Gilolo, which we thought we must be nearing, when we heard a dull roaring sound, like a heavy surf, behind us. In a short time the roar increased, and we saw a white line of foam coming on, which rapidly passed us without doing any harm, as our boat rose easily over the wave. At short intervals, ten or a dozen others overtook us with great rapidity, and then the sea became perfectly smooth, as it was before . . . . On reference to the old voyagers we find that these seas have been long subject to similar phenomena. Dampier encountered them near Mysol and New Guinea, and describes them as follows: "We found here very strange tides, that ran in streams, making a great sea, and roaring so loud that we could hear them before they came within a mile of us. The sea round about them seemed all broken, and tossed the ship so that she would not answer her helm. These ripplings commonly lasted ten or twelve minutes, and then the sea became as still and smooth as a millpond. We sounded often when in the midst of them, but found no ground, neither could we perceive that they drove us any way. We had in one night several of these tides, that came mostly from the west, and the wind being from that quarter we commonly heard them a long time before they came, and sometimes lowered our topsails, thinking it was a gust of wind. They were of great length, from north to south, but their breadth not exceeding 200 yards, and they drove a great pace. For though we had little wind to move us, yet these would soon pass away, and leave the water very smooth, and just before we encountered them we met a great swell, but it did not break.
>
> From *The Malay Archipelago* by A. R. Wallace, Harper & Brothers, Publishers, New York (1869).

In 1980, a major two-week experiment was conducted by the authors and their associates using the National Oceanic and Atmospheric Administration research vessel *Oceanographer*, which was on its way to the People's Republic of China for a cooperative research program off the Yangtze River. The experiment was designed to test the hypothesis that the striations were surface signatures of solitons generated by strong semidiurnal tidal flow over an unknown bottom feature near the southern rim of the Sea, which then radiated out into the interior, to be dissipated near the shore of Palawan Island bounding the northwest sector of the Sea. The experiment had three parts: (a) a satellite observation program using multispectral radiometers on the polar-orbiting satellites DMSP, NOAA-7, and Nimbus-7; (b) a set of current and temperature observations from a phased array of moored current meters and thermistors emplaced for two weeks at three sites ranging from the source region to halfway across the Sulu Sea (Fig 3b); and (c) a shipboard program using a variety of probes, including conductivity-temperature-depth profilers, a downward-looking acoustic echo sounder, and ship's X-band radar, the latter to observe the rough surface regions overlying the solitons. The experiment was supplemented by both theoretical analyses and stratified tank studies upon conclusion of the field phase. Detailed results will be published in early 1984.[12]

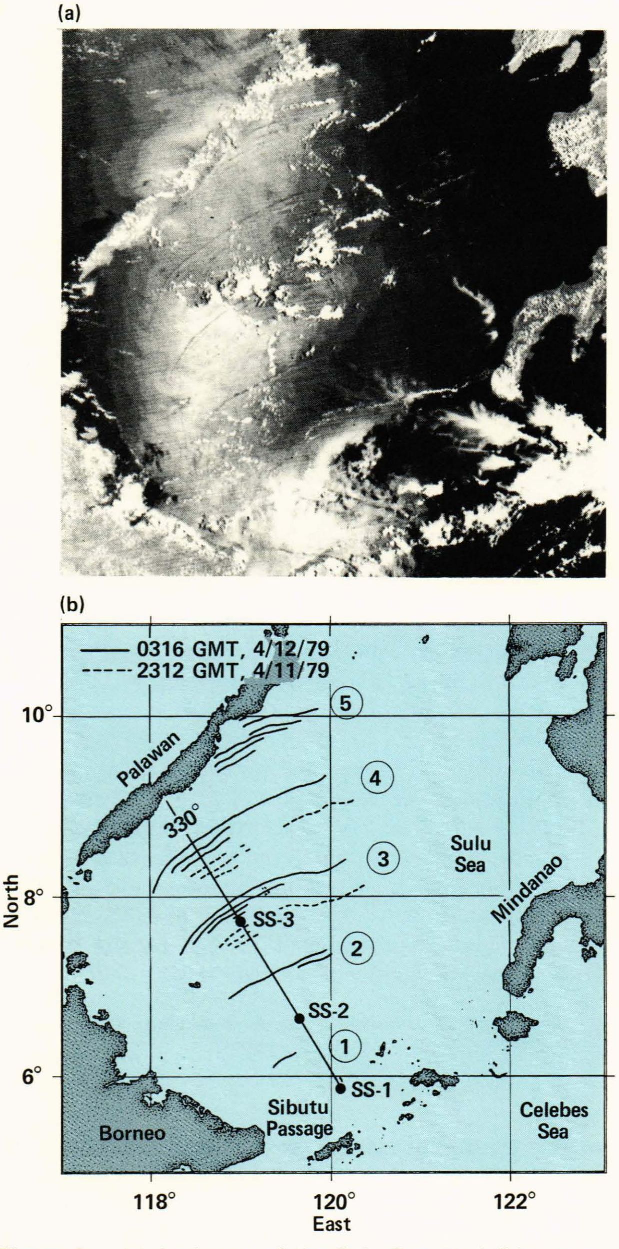


Figure 3 — (a) An image of the Sulu Sea at visible wavelengths, from the Defense Meteorological Satellite Program at 0316 GMT, April 12, 1979, showing surface striations overlying internal solitary waves, as observed in the sun glint. Five packets of solitons are seen spaced approximately 80 to 90 kilometers apart, each packet being generated during a strong ebb tide in the Sibutu Passage. (b) Line-drawing interpretation of striations suggesting region of origin and increasing wavelengths as waves cross the Sulu Sea in two days. Two DMSP images made 4 hours 4 minutes apart directly showed advance of two of the packets at a speed of 2.6 meters/second. Also shown are positions of the three elements, SS-1, SS-2, and SS-3, of a phased array of moored current meters used for direct measurement of soliton properties in May 1980.

Figure 4 is a several-hour segment of current measurements made at four depths from the mooring site SS-3 in the middle of the Sulu Sea (*cf* Fig. 3b). It shows a packet containing at least seven solitons propagating past the moorings as monodirectional pulses of current with speeds of up to 100 centimeters per second, or approximately 2 knots. The characteristic solitary shape is apparent in this very coherent wave packet. During the 11 days when the mooring actually acquired data, more than 13 such packets were observed at approximately 12.5 hour intervals, with the maximum amplitudes occurring near the peak of the fortnightly (14 day) tidal modulation.

Accompanying each of the internal solitons was a narrow stripe of breaking surface waves approximately 1000 meters wide, with amplitudes near 1 meter in a sea having a wave height of perhaps 30 centimeters, followed by a smoother-than-average region. Figure 5 shows a line of those whitecaps approaching the bow of *Oceanographer*, whose surface search ra-

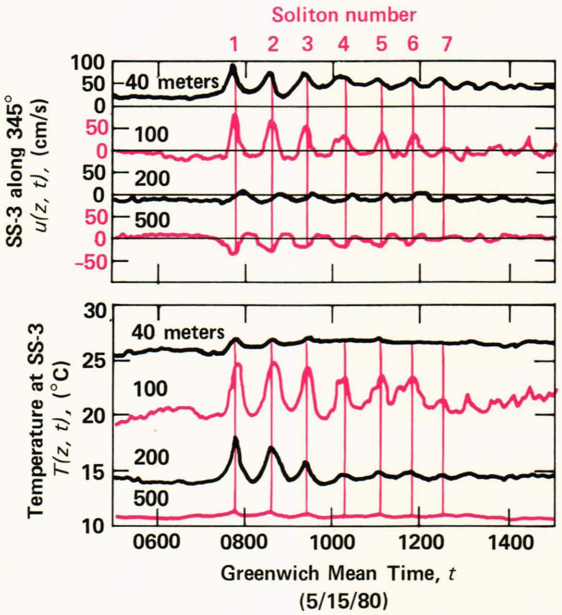


Figure 4 — Horizontal currents and temperatures at mooring site SS-3, 200 kilometers from the source, showing a packet containing more than seven solitons. Instruments at four depths were used. The phase reversal of currents between 100 and 500 meters depth reflects the cellular nature of circulation within a soliton. More than 13 such packets were observed during the two-week experiment.

Figure 5 — A photograph of the surface of the Sulu Sea as a soliton approaches research vessel *Oceanographer*. Breaking surface waves have approximately 1-meter amplitudes and are visible to satellite scanners and synthetic aperture radar, and to shipboard radar. The breaking waves are audible as they pass the ship.

dar clearly showed several of the rough stripes as regions of increased radar sea clutter. Using a combination of radar and visual observations, it was possible in one instance for the ship to follow a packet of solitons for over two days along a 330° sector from the region of their generation across the Sulu Sea to the reefs off Palawan Island, a distance greater than 380 kilometers.

It is these stripes of rough ocean surface that appear as striations in the satellite images. One such image obtained by LANDSAT in 1972 shows a striation (or crest) length in excess of 350 kilometers, a northern extreme at over 480 kilometers from the source region, a separation between solitons (wavelength) approaching 20 kilometers, and an implied lifetime of over 2½ days. These are indeed very large, long-lived oceanic phenomena.

The evidence that these excitations are truly solitons comes from analyses of the entire range of data obtained during and subsequent to the experiment, including, in particular, waveform and speed characteristics. A short discussion of the mathematical description will illustrate this. A deep-water, steady-state internal soliton has a downgoing vertical displacement of an isotherm of the form

$$\eta(x,z,t) = -\eta_0 W(z) \operatorname{sech}^2[(x-ct)/L] , \quad (2)$$

where the propagating horizontal shape factor is given by the square of the hyperbolic secant, with a horizontal scale of L and a nonlinear speed of c; the variation of amplitude with depth is described by $W(z)$. This equation is a solution to the nonlinear Korteweg-deVries differential equation in the asymptotic time-space limit and is illustrated schematically in Fig. 2. The nonlinear speed is dependent on peak amplitude η_0 approximately as

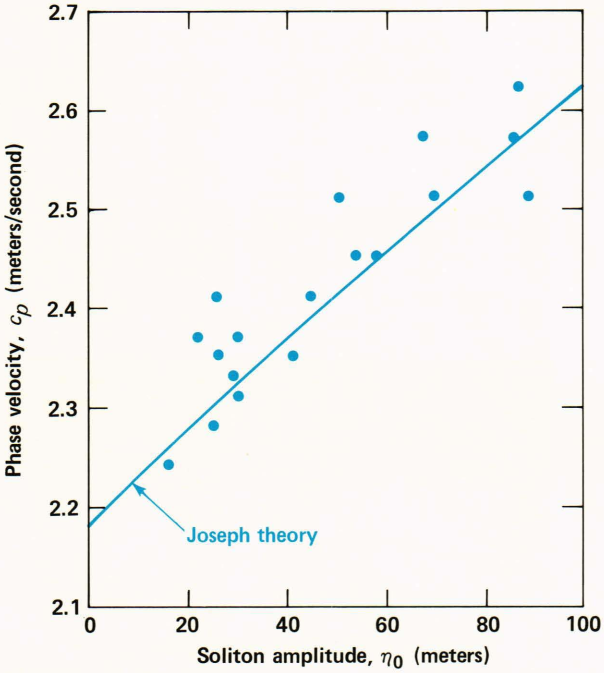


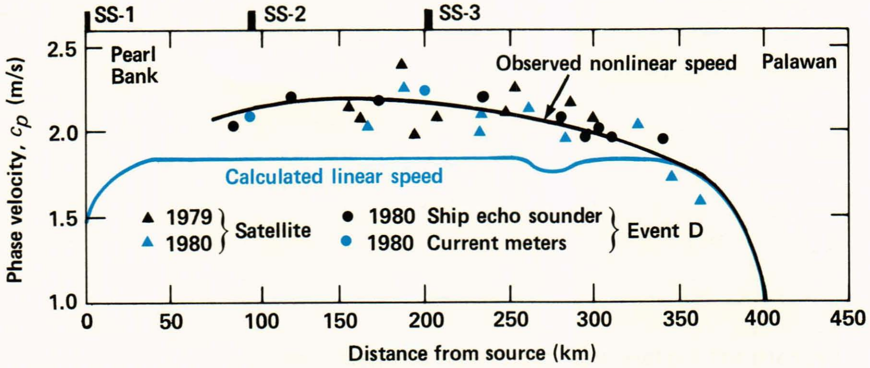
Figure 6 — The phase speed of 18 solitons versus their amplitude, as measured between pairs of current meter moorings; larger waves travel faster. The solid line is an *ab initio* calculation from the theory of Joseph[15], using actual environmental data. The good fit is evidence of the solitary character of waves.

$$c \cong c_0(1 + \alpha\eta_0) , \quad (3)$$

where α is a nonlinear coefficient dependent on density and depth. More generalized equations for soliton shape and speed in arbitrary water depth have been derived by R. I. Joseph.[15] Figure 6 shows speed-amplitude data for 18 solitons observed with current meters in the Sulu Sea, as compared with the Joseph theory evaluated using actual density measurements from the field program. The calculation is an *ab initio* one, with no adjustable parameters. Both data and calculation tend toward a small-amplitude speed, c_0, of 2.18 meters per second. The goodness of the fit clearly argues for the soliton nature of the waves. Similarly, the current and temperature waveforms fit the Joseph theory in a way that explains a high percentage of the signal variance.

Another data set (Fig. 7) shows the way soliton speed varies across the Sulu Sea, with data from three sources showing much congruence. Nineteen DMSP images from 1979 and 1980, similar to Figs. 3a and 3b, were analyzed for propagation speeds, which were inferred from distances between packets and times between zero crossings of the semidiurnal tides; the speeds are shown by triangles. Another data set on the figure comes from propagation delays between pairs of current-meter measurements and focuses on a particular packet of solitons identified as Event D (which was also the group followed across the Sulu Sea by *Oceanographer*); these are shown by

Figure 7 — The variation of soliton phase speeds across the Sulu Sea from satellite, ship, and current meters. Waves initially increase their speed as they form (*cf* Fig. 1), then decrease as they spread radially and lose amplitude. All three methods give similar results, indicating that waves propagate faster than at the calculated linear speeds (colored line) for most of their lifetime.



the colored circles at SS-2 and SS-3. The third data set, consisting of black circles, was obtained from Event D amplitudes observed with the acoustic echo sounder and converted to speeds using the Joseph equation. The agreement between these data sets is considered excellent for geophysical measurement. This figure shows two major tendencies: (a) the growth of a group of rank-ordered solitons out of the initial disturbance north of the sill at SS-1, which is offset by (b) the radial spreading and attendant diminution in amplitude as the mature solitary wave packet crosses the sea toward Palawan. Near the site of SS-3, the solitons are essentially fully evolved and at their maximum amplitude.

NUMERICAL SIMULATION

The Joseph solution to the governing evolution equation is a steady-state one describing a single isolated soliton in the asymptotic limit. For arbitrary times and positions, more general methods of analysis are appropriate, including inverse scattering theory, or numerical solutions to the full evolution equation as derived by Kubota *et al.*[16] The latter method has been used by Liu in conjunction with observations from the Sulu Sea experiment to predict the propagation of an observed thermocline depression in space and time between the current meter moorings at SS-2 and SS-3 (*cf* Fig. 3b).

Figure 8 shows the simulated space-time evolution of a displacement signal at 100 meters depth, in a coordinate system moving at a speed c of 2.35 meters per second, as it propagates across the Sulu Sea from SS-2 (time = 0) to SS-3 (12.5 hours later and 110 kilometers distant). The initial waveform data are from one partially evolved solitary wave observed at the intermediate mooring site. One sees in this sequence the change from an undulatory internal bore to a set of rank-ordered solitary waves, with the largest Fourier components moving from the back of the packet to the front; the leading edge of the packet moves at approximately 2.5 meters per second. An approximate correction has been applied to Liu's calculation for cylindrical spreading of the wave group as it crosses the Sea. This brings the amplitude excursions into good agreement with those observed at 100 meters (see Fig. 4, for example), as are the number of solitons, their wavelengths, and their propagation speeds. Improved calculations are under way to refine these results.

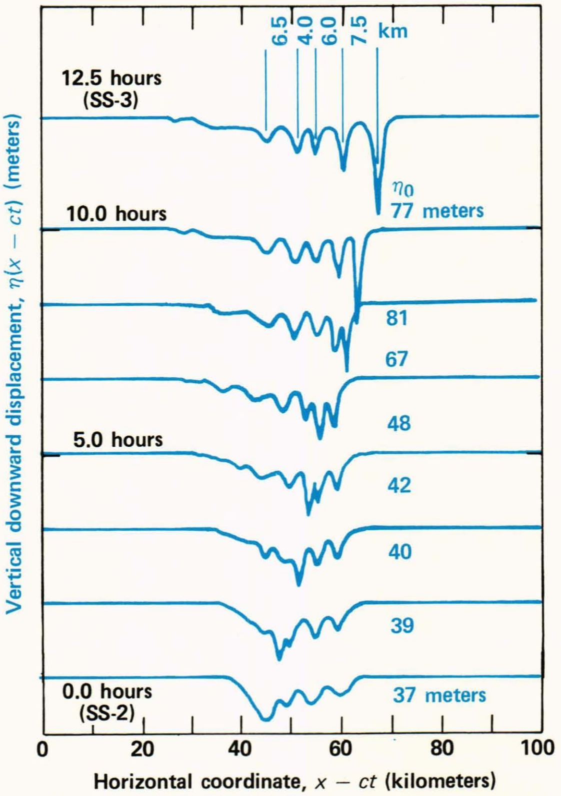


Figure 8 — A numerical simulation of the amplitude signal of a soliton packet propagating from mooring SS-2 to SS-3. Note how the solitons evolve from an undulatory bore into a rank-ordered sequence followed by a linear dispersive tail.[16] The solution is for the Kubota integro-differential equation with observed initial conditions and an approximate correction for radial spreading. The simulation is considered to be good.

SUMMARY

Internal solitary waves are among the largest and most coherent vertically varying signals in the sea.

Their special properties allow measurements to be made of a variety of types that, together with theoretical and numerical models, give unprecedented information on the dynamics of the phenomenon. There remain several interesting major problems related to the generation and dissipation processes and to the extraction of rotational energy of the earth-moon system and its deposition into small-scale oceanic motions via tides and internal wave generation. It is amusing to reflect that satellite imagery of the surface of the sea may be able to make a significant contribution to the understanding of events occurring beneath the surface that are driven ultimately by astronomical forces.

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