H.-P. PAO, ROBERT Y. LAI, and CHARLES E. SCHEMM

VORTEX TRAILS IN STRATIFIED FLUIDS

Visible atmospheric vortex trails caused by wind flow past certain islands have been observed by satellite photography. Laboratory experiments have shown that the collapse of the hydrodynamic turbulent wake produced by towing a three-dimensional body in stratified fluids produces a vortex pattern much like the two-dimensional von Karman vortex street seen in the atmosphere. The present study provides evidence that these phenomena are intimately related.

BACKGROUND

In the early 1960's, a new and interesting flow phenomenon was discovered in the earth's atmosphere by meteorological satellites: the mesoscale (~100-kilometer) vortex trails in the lee of certain islands, including Madeira, Jan Mayen, and Guadalupe (Baja California), to name only the three most significant locations. Satellite pictures (Fig. 1) show a range of mesoscale eddies downstream of the island of Madeira, whose cloud patterns bear a striking resemblance to the well-known von Karman street-like vortex trail caused by a steady wind. Figure 2 shows a chain of vortices on the lee side of the Arctic island of Jan Mayen, east of Greenland. Berger and Wille[1] have discussed the various early studies that attempted to explain these vortex patterns; these explanations were based largely on analogies with the two-dimensional von Karman vortex street. However, the shape of Madeira resembles a flat hump and is quite unlike a bluff body of even small aspect ratio. With this fact in mind, Pao and Kao[2] have proposed an explanation of the origin of these atmospheric vortex patterns. The experimental results on which their explanation is based are discussed later in this article.

In a recent review paper, Griffin[3] cited several examples wherein vortex streets were observed in the laboratory at relatively low Reynolds numbers (10^2 to 10^4). The Reynolds number is defined here as

$$R_D = \frac{(UD)}{\nu}, \quad (1)$$

where U is the speed of flow past the object, D is the diameter of the object, and ν is the kinematic viscosity of the fluid. The Reynolds number can be interpreted as the ratio of inertial force to viscous force in the equations of motion, and the fact that similar phenomena have been observed over nine orders of magnitude of Reynolds number (10^2 to 10^{11}) suggests that formation of these complex vortices is nearly independent of Reynolds number.

However, the formation of vortices in the wake of a body in stratified surroundings, while generally independent of Reynolds number, appears to be critically dependent on a second nondimensional parameter, the internal Froude number. The Froude number is defined by

$$F_D = \frac{U}{(ND)} \text{ where } N = \frac{1}{2\pi}\sqrt{-(g/\rho_0)(d\rho/dz)}, \quad (2)$$

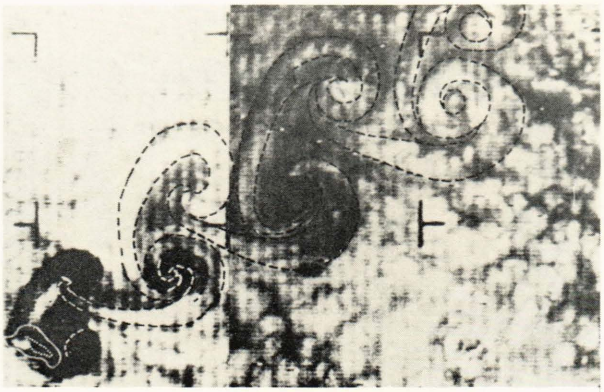


Figure 1 — Satellite photograph showing a cloud vortex downstream from the island of Madeira, which is at the lower left corner of the photograph.

Figure 2 — A chain of vortices in the clouds on the lee side of the Arctic island of Jan Mayen, east of Greenland. The photograph was taken from a National Oceanic and Atmospheric Administration (NOAA) satellite.

where N is the Brunt-Väisälä frequency in cycles per second (hertz). The Brunt-Väisälä frequency is the natural frequency of oscillation in a stably stratified fluid. Like the Reynolds number, the internal Froude number can be interpreted as the ratio of two competing forces in the equations of motion, in this case the inertial and buoyancy forces.

A review article by Lin and Pao[4] contains extensive discussions of turbulent wakes in stratified fluids, with particular emphasis on self-propelled body wakes.

EXISTENCE OF HORIZONTAL VORTICES

Vortex trails such as those observed in the atmosphere have also been observed in laboratory tank experiments. The wake behind a sphere in a thermally stratified fluid[5] at a time late of 285 seconds is shown in Fig. 3. The Reynolds and Froude numbers were 3125 and 11.9, respectively. The sphere was towed from left to right. Because of the flattened shape of the vortices in the horizontal plane, as illustrated by the side view in Fig. 3, they are usually referred to as horizontal or ''pancake'' eddies.

Essentially identical results are obtained[6] in salt-stratified water. The organized vortex pattern of Fig. 4 evolved from the fully turbulent wake of a towed sphere at an approximate time late of 2 minutes. For moderately high Froude numbers ($F_D > 10$) and larger Reynolds numbers ($R_D > 3 \times 10^3$), the horizontal vortices begin to form after about 10 Brunt-Väisälä periods.

Figure 5 shows the top view of a wake created in a stratified tank by flow past a stationary ellipsoid. The Froude number ($F_D = 3.9$) is relatively low. Horizontal vortices start to form at a location about eight body diameters downstream of the body. The vortex trail shown in Fig. 5 bears a striking resemblance to the satellite photographs in Figs. 1 and 2, the only difference being that the cloud vortex trail appears to form immediately behind the island, whereas the vortex pattern in the wake of the ellipsoid forms several body diameters downstream. Based on the width of the island normal to the wind direction, Pao and Kao[2] have obtained an estimated Reynolds number of 10^{11} for the atmospheric flow of Fig. 1. From meteorological data at the time of the satellite photographs, Moll[7] has shown that the atmosphere beneath the cloud layer was stably stratified. Assuming a typical atmospheric value of 5 minutes for the Brunt-Väisälä period and using the height of the mountainous island as the characteristic length scale, D, leads to an estimated Froude number between 0.5 and 1.0.

It is interesting that the distance of the initial vortex formation behind the body depends upon the magnitude of F_D. For high Froude numbers, this distance is usually several hundred body diameters. Although no laboratory data are available for $F_D < 2$, it can be inferred from the pattern of the cloud trails that, for very low Froude numbers, the vortex is likely to form immediately behind the body.

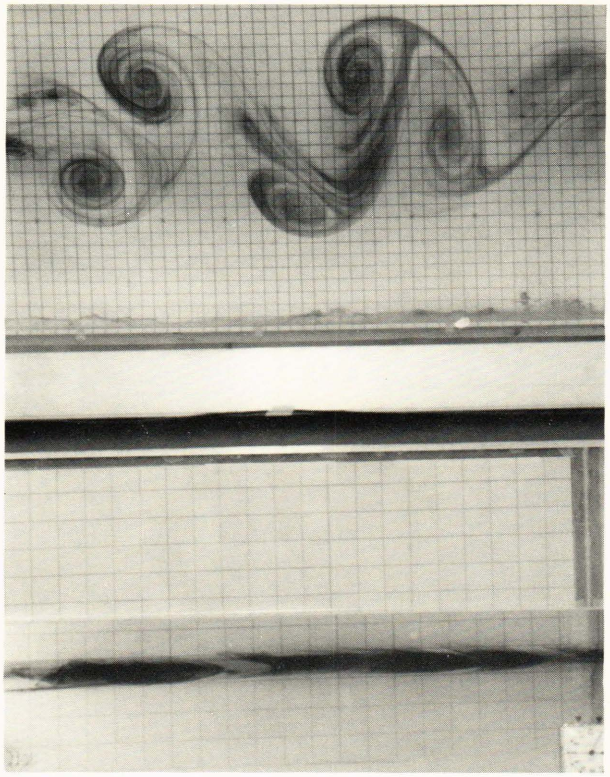


Figure 3 — Vortex structure in the wake of a sphere towed through thermally stratified water. The upper photograph shows the vortex trail in the horizontal plane at a time late of 285 seconds ($Nt = 21.4$) after the passage of the sphere. The lower photograph shows a side view of the relatively narrow vertical extent of the wake. The grid in the background is 1 × 1 inch in the upper photograph and 2 × 2 inches in the lower photograph. Sphere diameter, 2.25 inches; speed, 2.0 inches per second; N, 0.075 hertz; R_D, 3125; F_D, 11.9.

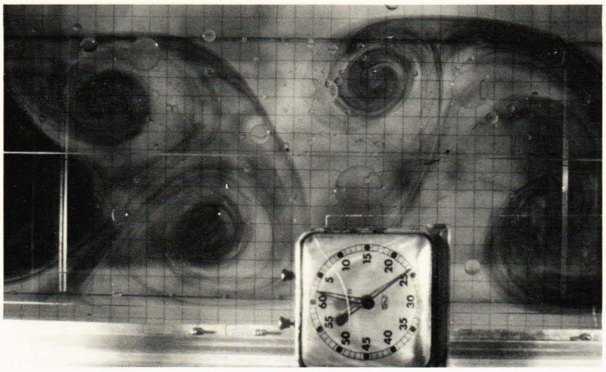


Figure 4 — Vortex pattern in the wake of a sphere towed through salinity-stratified water. The photograph shows the vortex trail viewed from the top at a time late of 124 seconds ($Nt = 11.2$) after the passage of the sphere. The grid in the background is 1 × 1 inch. Sphere diameter, 2.25 inches; speed, 2.75 inches per second; N, 0.090 hertz; R_D, 4300; F_D, 13.6.

Tank experiments have shown that the flow is shed three-dimensionally for all Froude numbers above 2. In the absence of laboratory data, one is uncertain whether the flow will be shed two-dimensionally

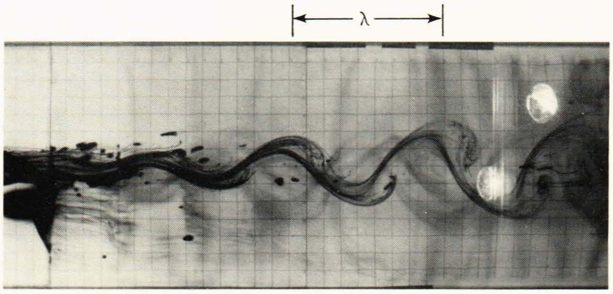


Figure 5 — Top view of the wake behind an ellipsoid in a flowing stratified fluid. The ellipsoid is just beyond the left edge of the picture. The flow direction is from left to right. The grid in the background is 1 × 1 inch. Flow speed, 0.8 inch per second; ellipsoid diameter, 2 inches; ellipsoid length, 8 inches; N, 0.065 hertz; R_D, 1100; F_D, 3.9.

from a three-dimensional bluff body for a very low Froude number.

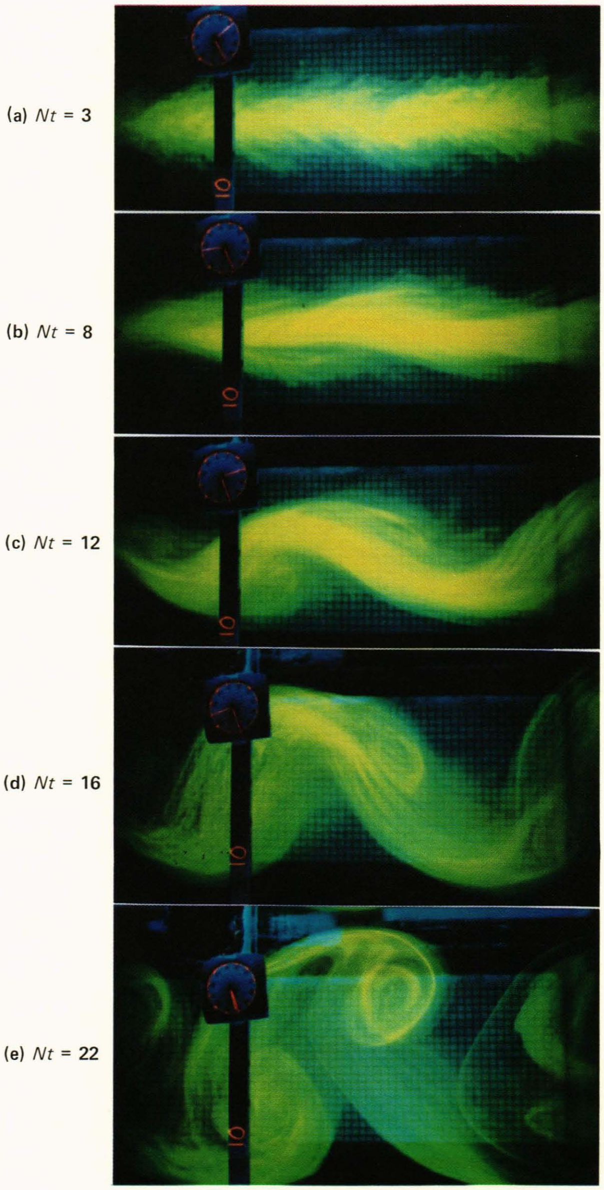
Horizontal vortices also form in the wake of a slender body in a stratified fluid[8] (Fig. 6). The late-time vortex pattern in Fig. 6e bears a striking resemblance to the vortex patterns in the wake of a sphere that were shown in Figs. 3 and 4. Vortex patterns similar to those behind towed bodies also develop in the wakes of self-propelled bodies.[4]

From these tank observations, we conclude that horizontal vortices form in the wakes of bluff bodies, slender bodies, and self-propelled bodies for $2 \leq F_D \leq 505$ and $600 \leq R_D \leq 10^5$. We conclude from satellite photographs that such horizontal vortices can exist for Froude number of one or less and for Reynolds numbers as high as 10^{10} to 10^{11}

SIMILARITY OF HORIZONTAL VORTICES

As we have observed, the patterns of these horizontal vortices are indeed very regular. Moreover, for high Froude numbers, wake evolution is similar for different body shapes (bluff and slender) and appears to be independent of whether the body is towed or self-propelled. Wake evolution in stratified fluids can be described as consisting of four stages:[4,8] (a) growth, (b) modulation and spreading, (c) meandering, and (d) vortex. These stages of development are illustrated in Fig. 6. Figure 6a shows the modulation and spreading stage, Figs. 6b and 6c the meandering stage, and Figs. 6d and 6e the vortex stage. From the available laboratory data, it is found that the characteristic wavelength of meandering is very close to the spacing of vortices in the subsequent vortex stage. The dimensionless spacings λ/d are between 5 and 7 for towed spheres, between 7 and 11 for towed slender bodies, and between 3 and 5 for self-propelled slender bodies. These results hold for a wide range of Froude and Reynolds numbers.

Some questions may arise: What is the generation mechanism of these horizontal vortices? Why has such regularity resulted from a seemingly chaotic initial turbulent wake?


(a) $Nt = 3$

(b) $Nt = 8$

(c) $Nt = 12$

(d) $Nt = 16$

(e) $Nt = 22$

Figure 6 — Dye visualization of the wake of a towed slender body in a stratified fluid. The grid in the background is 1 × 1 inch. Towing speed, 2.0 feet per second; body diameter, 3 inches; body length, 3.6 inches; N, 0.129 hertz; R_D, 5×10^4; F_D, 62.

GENERATION OF HORIZONTAL VORTICES

Mechanism for Bluff Bodies

A clue to the regularity of the late-time horizontal vortices is provided by the existence of large-scale coherent vortex structures in the early wake. Pao and Kao[6] have proposed that the early wake of a sphere consists of a double-helical vortex loop pattern. As this flow pattern evolves in time, the stratified wake collapses vertically and spreads horizontally, with the motions attenuated more rapidly in the vertical than in the horizontal direction. Under this hypothesis, the late wake vortex pattern is a remnant of the early wake's helical structure. The horizontal spacing between vortex loops in the early wake is of the same magnitude as the spacing between vortices in the late wake.

Pao and Kao's vortex structure model is supported by the measurement of Fuchs, Mercker, and Michel,[9] whose two-point cross correlation reveals the existence of a well-ordered traveling turbulence pattern. They also found an antiphase relationship of signal in the wake that is consistent with the antisymmetry of the double-helical vortex loop pattern.

More recently, Perry and Watmuff[10] have demonstrated the existence of large-scale coherent structures in the wakes of three-dimensional blunt bodies (oblate ellipsoids) at Reynolds numbers of ~32,500. By using a "flying hot-wire" apparatus in their experiment and describing the vortex shedding cycle by phase-averaged vector fields, they were able to produce an animation of large-scale motions. These large-scale coherent structures retain their identity for long streamwise distances. Figures 7a and 7b show the phase-averaged vector fields for the near wake and the far wake, respectively, as seen by an observer moving with the coherent structures. The far-wake region is about 20 equivalent body diameters behind the body.

It is interesting that the large-scale coherent structure shown in Fig. 7 is consistent with the early hypothesis proposed by Pao and Kao.[6] The schematic representation of the vortex configuration of the Pao and Kao model is reproduced in Fig. 8. A section cut through the central plane of the wake axis in the top view of Fig. 8 actually yields a flow pattern almost identical with that shown in Fig. 7a.

The fact that these large-scale coherent structures retain their identity for long streamwise distances also confirms an earlier prediction by Pao and Kao[6] that the spacing between vortex loops in the early wake is of the same magnitude as the spacing between vortices in the late wake.

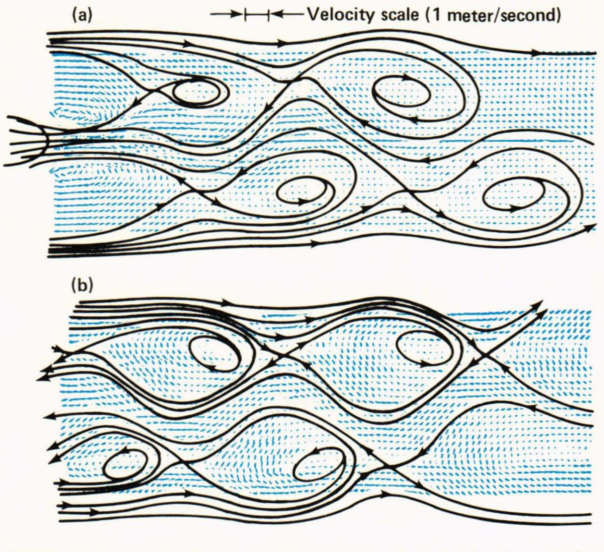
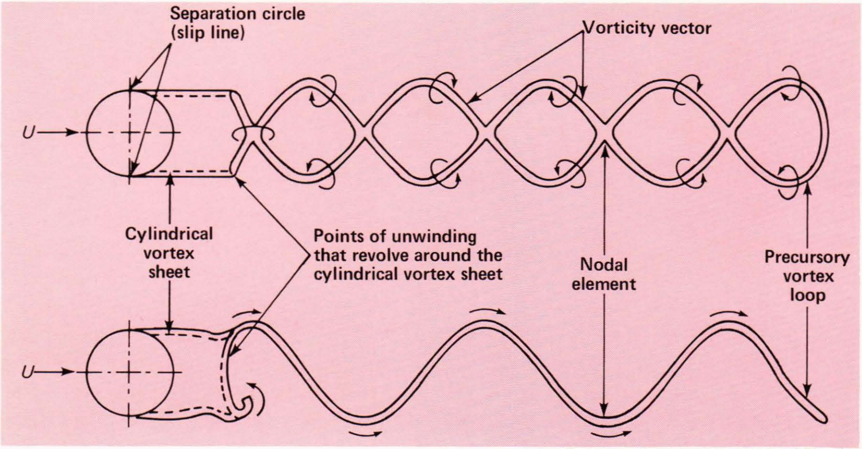


Figure 7 — Large-scale coherent structures in the wake behind an oblate ellipsoid as described by the phase-averaged vector fields: (a) near wake; (b) far wake (from Ref. 10).

The sequence that leads to the generation of the late-wake horizontal vortices in a turbulent wake of a bluff body is now quite clear. First, the large-scale coherent structure that exists in the early wake remains as an organizing agent throughout the evolution of the wake as stratification suppresses large-scale turbulence and, possibly, leads to meandering of the wake. Second, if the random turbulence has not destroyed the large-scale coherent structure, horizontal vortices will form. The nodal elements in the double-helical vortex loop structure of the early wake become vortex centers in the late wake.

Critical Froude Number

The preceding discussion of a possible generation mechanism for horizontal vortices contained no mention of the role of varying degrees of stratification on eddy formation. We now consider whether there exists an upper limit of Froude number above which no horizontal vortices will develop in the late wake. We



Courtesy American Institute of Physics

Figure 8 — Schematic representation of the double-helical vortex configuration in the wake of a sphere (from Ref. 6).

conclude that such a critical Froude number in all likelihood does exist and that its value for a bluff body is about 160.

The basic cause of late-wake vortices is the large-scale coherent structure present in the early wake. In homogeneous fluids, the coherent structure can exist only within a certain distance of a bluff body; beyond that limit, the coherent structure may be destroyed by random turbulence. In stratified fluids, however, random turbulence may be suppressed by gravitational forces.

The two competing mechanisms present in a stratified turbulent wake may be characterized by two time scales. One is the residence time, t_R, defined as the period of time during which the coherent structure remains and after which it is destroyed by random turbulence. The other is the response time, t_S, defined as the time period within which wake turbulence overwhelms the effects of stratification and after which stratification becomes the dominant influence on wake evolution. If $t_S > t_R$, then at times late between t_R and t_S the coherent structure will be destroyed by random turbulence before stratification can take effect. Consequently, horizontal vortices will not form, even at late times. On the other hand, if $t_S < t_R$, then stratification will significantly influence a wake turbulence in the period between t_S and t_R. In this case, coherent structures may be preserved to late times, and horizontal vortices will form.

An expression for the critical Froude number may be derived by referring to the simple schematic in Fig. 9. If L is the distance behind the bluff body within which coherent structures remain, then the residence time is given by $t_R = L/U$. If $L = a_1 D$ and $t_S = a_2 N^{-1}$, where a_1 and a_2 are constants, then the critical condition is

$$\begin{aligned} t_S &= t_R, \text{ or} \\ a_2 N^{-1} &= (a_1 D)/U, \text{ or} \qquad (3) \\ U/(ND) &= a_1/a_2 \ . \end{aligned}$$

Using $a_1 = 40$ and $a_2 = 0.25$, we obtain a value for the critical Froude number of about 160. The choice of values for a_1 and a_2 was based on assumptions that the vortex structure remains coherent over a downstream distance of 40 body diameters and that stratification has little influence in the first quarter of the Brunt-Väisälä period.

It was to confirm the existence of such a critical Froude number that the aforementioned laboratory experiments of towed spheres were performed. For values of Froude number greater than 300, no horizontal vortices were observed at late times. The above derivation of the critical Froude number was given most recently by Pao and Kao.[11]

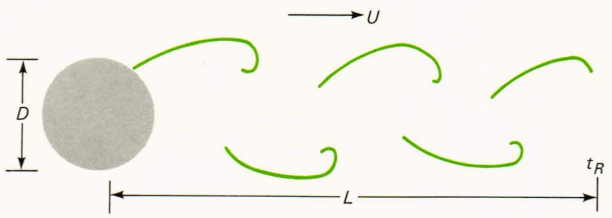


Figure 9 — Schematic drawing of a wake behind a bluff body.

Mechanism for Slender Bodies

The generation mechanism for late-wake horizontal vortices discussed in previous sections is based mainly on the existence of large-scale coherent structures in the early wake and the effect of stratification in preserving these coherent structures. However, no vortex shedding has been detected immediately behind a slender body.

Sato and Okada[12] have studied the stability and transition of a wake behind an axisymmetric slender body in a wind tunnel. Their results show that, for a certain frequency range, the small-amplitude velocity fluctuations are amplified as they travel downstream. Phase measurements in the azimuthal and axial directions indicate that the line of constant phase forms a helix and not a discrete closed loop. This suggests the existence of a helical vortex structure in the wake.

Based on the experimental results of Sato and Okada, one can estimate the spacing λ between two adjacent vortices or, equivalently, the pitch length of the helix. The dimensionless spacing λ/d is found to be 7.3, a value within the range of spacings between vortices in slender-body wakes at late times.

Detailed measurements to determine whether large-scale coherent structures exist in the early wake of a slender body have yet to be made, but the results of Sato and Okada[12] suggest that the existence of such coherent structures in the early wake is highly probable. The agreement that they found between helical structure pitch and the spacing between late-wake vortices lends further credence to the proposed generation mechanism. Thus, we conclude that the generation mechanism for the late-wake horizontal vortices behind bluff bodies is also applicable to slender bodies.

In recent tank experiments at APL with a towed slender body, no late-wake horizontal vortices were observed when the internal Froude number exceeded 600. This finding suggests that a critical Froude number also exists for slender-body wakes.

Mechanism for Self-Propelled Bodies

As mentioned earlier, the late-wake vortex patterns behind bluff, slender, and self-propelled bodies are all very similar. However, the late-wake vortex patterns from self-propelled bodies are not so regular as those from towed bodies. Moreover, the spacing between vortices is considerably smaller than for equivalent towed bodies. Two possible reasons for these differences are offered. The propeller of the self-propelled body may alter the length scale and other characteristics of the large-scale coherent structures in the early wake. The differences could also be due to fundamental differences between momentumless and drag wakes. In either case, the same basic generation mechanism seems to be operating for towed and self-propelled bodies.

CAUSE OF MEANDERING

It is observed experimentally that wake meander is a distinct stage in the evolution of the stratified turbulent wake at moderately high Froude numbers. In wakes of self-propelled slender bodies, meander usually begins at $Nt = 4$ (t is time in seconds), after the cross section of the wake has become flattened as a result of vertical collapse and horizontal spread of the stratified wake. From what has been learned concerning the generation mechanism, it appears likely that the large-scale coherent structure in the early wake leads to shear instability, which is manifest in wake meander. Horizontal meander is simply the preferred mode of instability.

EFFECT OF AMBIENT SHEAR

In a recent laboratory study, Pao and White[8] have shown that shear has a significant effect on the later stages of wake development. In their experiments, streamwise vertical shear was produced in the tank by the method of selective withdrawal. In a stratified shear flow, the important parameter governing the stability of flow is the Richardson number. It is defined by

$$R_i = \left(\frac{2\pi N}{\partial u/\partial z}\right)^2 , \tag{4}$$

where $\partial u/\partial z$ is the ambient vertical shear, which is usually the source of flow instability. The Richardson number can be interpreted as the ratio of two counteracting factors in the flow instability mechanism. The Brunt-Väisälä frequency, N, is a measure of stabilizing effect, whereas $\partial u/\partial z$ represents the destabilizing effect.

Figure 10 is the top view of a self-propelled body wake in a sheared stratified flow. The direction of flow in the photograph is from top to bottom. This selective withdrawal arrangement resulted in a higher shear near the lower portion of the photograph than near the upper portion. Thus, at $Nt = 13$, horizontal vortices are observed in the low-shear region. By $Nt = 19$, the vortices have begun to disappear, starting from the high-shear region. It is concluded that longitudinal shear either has destroyed existing horizontal vortices at late time or has effectively prevented vortex formation altogether.

The absence of late-wake horizontal vortices in stratified shear flows may be attributed to any of three possible mechanisms:

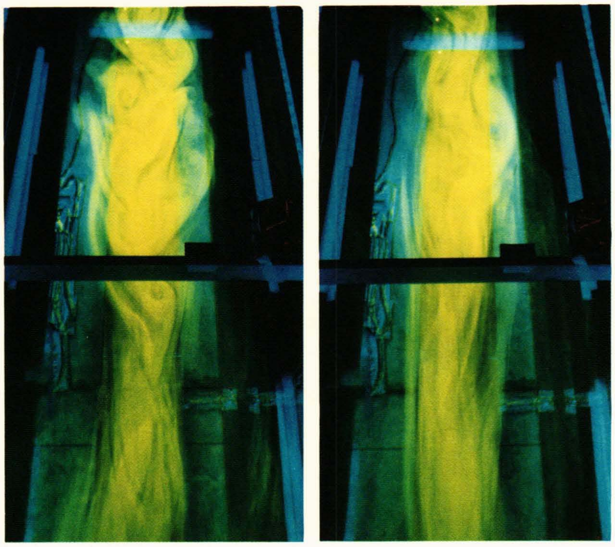
1. From a kinematic viewpoint, layers at different depths move with different speeds in shear flow. Therefore, the dye pattern as seen from above could appear to be dispersed or disorganized.
2. Vertical shear tilts the vertical vorticity vector while stratification suppresses the vertical motion. This would tend to align the vorticity vector with the stream surface, which is essentially in the horizontal plane. In this case, the vertical component of vorticity would gradually diminish, resulting in the disappearance of the vortex pattern at late times.
3. Shear feeds energy into turbulence, which, in turn, may quicken the destruction of the large-scale coherent structures in the early wake. Since large-scale coherent structures are believed to be the cause of the horizontal vortex formation in the late wake, their destruction would suppress formation of horizontal vortices.

Based on the foregoing reasoning, one may tentatively conclude that both streamwise and spanwise vertical shear have the same effect. They either prohibit the formation of horizontal vortices, or they destroy horizontal vortices in the later stages of wake development.

CONCLUSIONS

The principal conclusions that can be drawn at the present time are:

1. Density stratification is the primary reason for the appearance of cloud vortex trails over islands in the ocean.



(a) $Nt = 13$ (b) $Nt = 19$

Figure 10 — Dye visualization of the wake of a self-propelled body in a sheared stratified flow (top view). Speed, 2.0 feet per second; body diameter, 3 inches; body length, 36 inches; N, 0.127 hertz; R_D, 5×10^4; F_D, 63; (R_i), 2.3.

2. Large-scale coherent structure in the early wake is the cause of wake meander, which, in turn, leads to the formation of horizontal vortices at late times.
3. There exists a critical Froude number above which no horizontal vortices will develop in the late wake.
4. Ambient shear can effectively prevent formation of or destroy late-wake horizontal vortices.

Thus, we have essentially established the origin, existence, and destruction of these atmospheric vortex trails in the lee of certain islands. Similar mechanisms are also operative in the wake of a three-dimensional body in a stratified fluid.

REFERENCES

[1] E. W. Berger and R. Wille, "Periodic Flow Phenomena," *Annu. Rev. Fluid Mech.* **4**, 313-340 (1972).

[2] H. -P. Pao and T. W. Kao, "On Vortex Trails Over Ocean Islands," *Atmos. Sci.* (Meteorological Society of the Republic of China) **3**, 28-38 (1976).

[3] O. M. Griffin, "Observations of Vortex Streets and Patterns in the Atmosphere, in the Oceans, in the Laboratory," Symposium on Vortex Flows, ASME Winter Annual Meeting (1980).

[4] J. T. Lin and Y. H. Pao, "Wakes in Stratified Fluids," *Annu. Rev. Fluid Mech.* **11**, 317-338 (1979).

[5] T. W. Kao and H. -P. Pao, "Note on the Flow of a Stratified Fluid Over a Stationary Obstacle in a Channel," *J. Geophys. Astrophys. Fluid Dynamics* **10**, 109-114 (1978).

[6] H. -P. Pao and T. W. Kao, "Vortex Structure in the Wake of a Sphere," *Phys. Fluids* **20**, 187-191 (1977).

[7] H. G. Moll, "Die Atmosphärische Umströmung Maderas," *Betir. Phys. Atmos.* **44**, 227-244 (1971).

[8] H. -P. Pao and B. L. White, "Experiments on the Turbulent Wake of an Axisymmetric Body in a Stratified Shear Flow," Joint ASME/ASCE Mechanics Conference, Boulder, Colo. (June 1981); also JHU/APL STK-81-004 (1981).

[9] H. V. Fuchs, E. Mercker, and U. Michel, "Large-Scale Coherent Structures in the Wake of Axisymmetric Bodies," *J. Fluid Mech.* **93**, 185-207 (1979).

[10] A. E. Perry and J. H. Watmuff, "The Phase-Averaged Large-Scale Structures in Three-Dimensional Turbulent Wakes," *J. Fluid Mech.* **103**, 33-51 (1981).

[11] H. -P. Pao and T. W. Kao, "On the Existence of Large-Scale Coherent Structure in the Wake of a Towed Body in a Stratified Fluid," Joint ASME/ASCE Mechanics Conference, Boulder, Colo. (June 1981).

[12] H. Sato and O. Okada, "The Stability and Transition of an Axisymmetric Wake," *J. Fluid Mech.* **26**, 237-253 (1966).