PERFORMANCE in ORBIT of the TRIAD DISTURBANCE COMPENSATION SYSTEM

R. E. Jenkins

The performance of the TRIAD DISCOS system in orbit has been monitored using telemetry data and doppler tracking data. Early in the satellite life, the telemetry data showed that the system was operating normally, almost exactly as in the design specifications. The doppler data were used to test the orbit predictability with the DISCOS system in operation. Accurate prediction was attained for longer than a week compared with the present 24-hour capability. Experiments with a prediction span up to two months showed that the drag and radiation forces were being entirely eliminated and that the DISCOS self-bias force was less than $10^{-11}g$.

Introduction

TO ANSWER THE QUESTION: "HOW WELL WERE the DISCOS design goals met?," both the telemetry data and the doppler tracking data from TRIAD were used. The telemetry data were used to confirm that the DISCOS control loop was stable, that the ball (proof mass) was in free flight in the cavity, and that the thruster activity was conforming to expected levels. The tracking data were used to determine the effect of the DISCOS device on the satellite orbital motion.

With the DISCOS device operating properly, the satellite feels no effects of solar radiation pressure or atmospheric drag forces. In the theory for satellite motion used in tracking and orbit prediction, these two forces are the ones most difficult to model and the ones which ultimately limit the prediction accuracy at medium altitude (nominal altitude of navigation satellites is 1100 km). With these forces eliminated, the satellite flies a purely gravitational orbit which, in principle, is predictable. In practice, however, various factors limit the predictability of the orbit. Some of these are, for example: tracking data precision, uncertainties in the earth's gravity field, and simply the numerical or analytical limitations in solving the satellite equations of motion.

An important factor in the TRIAD tracking is internal force biases between the DISCOS proof mass and the main body. Such biases can arise from various sources, and were carefully considered in the satellite design. Any force exerted on the proof mass by the main body will perturb the proof mass orbit, and the DISCOS system will thrust the main body to follow. The orbital response to such a force depends a great deal on its direction relative to the satellite velocity vector.

It is convenient to resolve perturbing forces into components in the "local vertical-orbit plane" system: along-track, cross-track, and radial. With a gravity-gradient, three-axis stabilized satellite such as TRIAD, these components correspond to satellite body-fixed directions, with the along-track direction along the satellite velocity vector. The satellite motion is extremely sensitive to a force in the along-track direction, since this force affects the magnitude of the satellite velocity which in turn changes the period. Small forces normal to the satellite velocity tend to rotate the velocity vector rather than change its magnitude, and hence they affect the orbit geometry rather than the period. Cross-track and radial forces smaller than 10^{-9}g have a nearly imperceptible effect on the satellite orbit. On the other hand, by observing the satellite over a sufficiently long arc, an along-track force as small as 10^{-12}g can be easily detected.

A goal in the satellite design was to keep the along-track self-bias force no larger than 10^{-11}g. As will be seen from the tracking results presented in this paper, that remarkable goal was exceeded. It will also be seen that the overall goal of increasing the orbit predictability from one day to one week was exceeded by a wide margin. In nearly all respects the DISCOS performed faultlessly for about ten months, until it was commanded off in August 1973.

Telemetry Data

The unfortunate failure of the telemetry system after one month in orbit prevented the accumulation of a large body of valuable atmospheric density data. However, the results that were obtained were sufficient to verify the proper performance of DISCOS.[1]

The telemetered DISCOS data included:

1. Vector proof mass position in the cavity (both coarse and fine scale).
2. Accumulated, commanded on-time in every four-minute interval for each of the six thrusters.
3. A "tell-tale" to indicate contact between ball and cavity.

The actual thruster on-time can differ slightly from the commanded on-time because of inevitable minor imperfections in the system. Similarly, the exact total impulse delivered for a given valve on-time depends on things such as the valve action and the fuel pressure. An in-orbit calibration is needed to relate the delivered impulse to the telemetered on-time, particularly when the fuel pressure is not known.

The nature of the telemetry failure was such that the wall contact indicator luckily was not lost. Throughout the time period that the DISCOS was operating, this tell-tale indicated no ball contacts with the cavity other than those purposely caused as part of the experiment.

The stability of the control system was established immediately after the satellite was placed in orbit and the ball "uncaged." The ball-capture sequence is shown in Fig. 6 in the article in this issue by J. Dassoulas. The thrusters were activated about a minute after the ball was uncaged and the capture transient disappeared after about four minutes.

A sample of the ball position data for all three components of displacement during normal operation is shown in Fig. 1. By fitting second-order polynominals to the position data during normal operations, the acceleration of the ball, relative to the satellite, can be determined. In the absence of ball interaction with the satellite, this acceleration is entirely due to the surface forces acting on the satellite. This is a potential source of high-precision aeronomy data, particularly when the satellite is in the dark.[2] From the velocity discontinuities caused by thruster action, an in-orbit calibration of the thrusters can be obtained. In the failure of the telemetering system, however, the ball position data could not be obtained and this potentially large body of valuable aeronomy data was lost.

In the analysis of the integrated thruster on-times, it was fortunate that the data contained a day (304) when the sun was nearly normal (81°) to the orbit plane. For this particular date, the radiation pressure force is nearly pure cross-track and orthogonal to the drag force. With this geometry, the cross-product of the radiation force and drag force is in the radial direction. A resolution of the thruster on-time data in these three directions is shown by the histogram plot in Fig. 2. The top curve (S) is the component along the satellite-sun line, the center curve (V) is the component along the satellite velocity vector, and the bottom curve (N) is the component along the "cross-product" direction. The top curve shows radiation pressure, the center curve shows drag mixed with a small along-track radiation force component, and the bottom curve shows the force normal to the plane determined by the other two force vectors. As mentioned, for this special case this is nearly the radial direction.

The extreme values on the ordinate (200 msec) correspond to a total impulse of 170 dyne-seconds. There are several things apparent from the data:

1. The drag force is, on the average, only ¼ to ⅕ as large as the radiation pressure force.
2. The along-track force has a strong orbital frequency component.
3. Surprisingly, there is a rather large radial bias force.

---

[1] A. Eisner and R. Yuhasz, *A Flight Evaluation of the DISCOS System on the TRIAD Satellite,* APL/JHU TG 1216, Jul. 1973.

[2] T. Potemra, Private Communication; also Internal APL Memorandum.

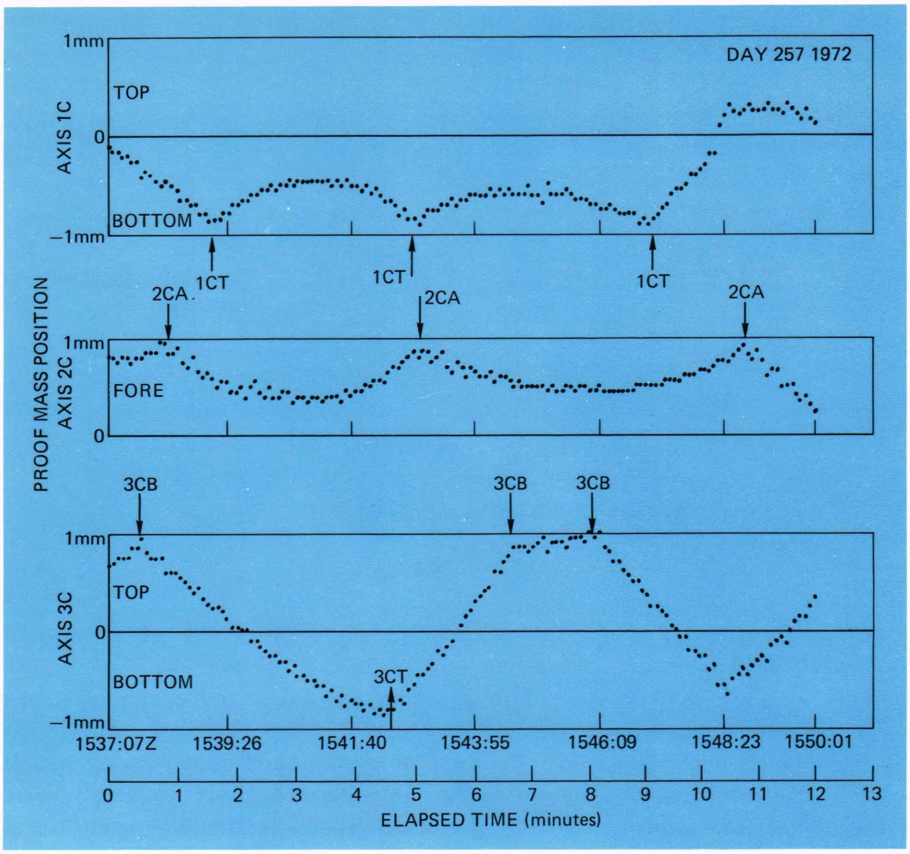

Fig. 1—DISCOS proof mass position, day 257.

The first two observations are consistent with the radiation pressure model, with current atmospheric density models, and with the fact that the orbit is polar.

The third observation triggered an intensive search for the source of the bias. The most likely cause is: the center of mass of the entire satellite does not lie at the center of the proof-mass cavity. As a result, the proof mass and the satellite are in different orbits. To follow the proof mass, a thrust component along the vertical is required due to the gradient in the earth's field. The most probable origin of this bias is a 1 cm maladjustment in one of the boom lengths. (The boom lengths were to have been adjusted in orbit, and had the telemetry system not failed, this bias would have been removed.)

A theoretical computation of the radiation pressure was used with the top curve to derive a calibration relationship between telemetered on-time and in-orbit thrust. From this calibration, the radial acceleration bias was computed to be about 4×10^{-9}g.

As a further test, models of the drag force, radiation pressure, and radial bias were used to compute the three acceleration components and compare them with the data. The results of the comparison are shown in Fig. 2 for the day 304 data, and in Fig. 3 for the day 257 data. The computed forces are the solid lines and the thruster on-times are the histogram format. In the day 257 data, the satellite-sun relative geometry is appreciably different, and our three force "directions" (S, V, and N) are no longer near-orthogonal. Thus, the forces in these directions become a mixture of the drag, radiation pressure, and radial bias, rather than being nicely separated as on day 304. The data show good agreement with theory, and the calibration achieved on day 304 has remained valid.

Tracking Experiments

The objectives of the TRIAD tracking experiments were to test the satellite orbit predictability, and to determine the level of the self-bias force present in the DISCOS unit. To perform the experiments and also to provide a continuous span of tracking data over one complete revolu-

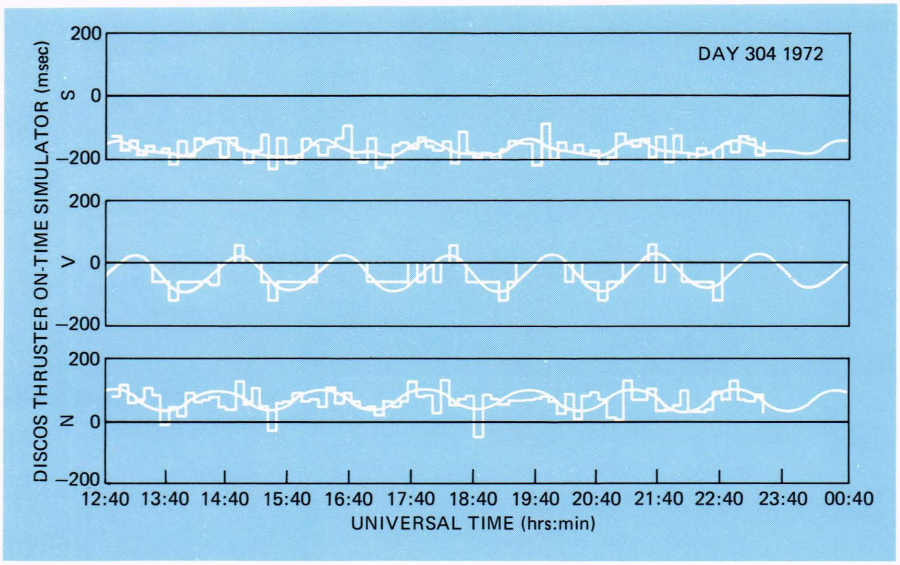

Fig. 2—Thruster on-time data, day 304.

tion of perigee for future geodesy purposes, the DISCOS system was kept in the active state without interruption for five months. This period began on October 18, 1972 and ended on March 13, 1973. During that time the proof mass did not touch the cavity wall, so the effects of drag and radiation pressure were completely eliminated. The satellite doppler data were continuously recorded by the TRANET system of thirteen worldwide stations. They averaged about 35 usable doppler passes per day during the five-month span. A doppler pass consists of about 15 minutes of continuously recorded doppler data as the satellite passes within view of a receiving station.

The experiments to determine the orbit predictability consist of first tracking the satellite over some time span, and then comparing doppler data to the satellite ephemeris (position versus time) predicted from the tracked orbit. Before presenting results, a description will be helpful on what is meant by "tracking" and on how the accuracy of the predicted ephemeris is measured.

The tracking process fits the theoretical satellite motion to the data in a group of doppler passes taken over some span of time. The (least-

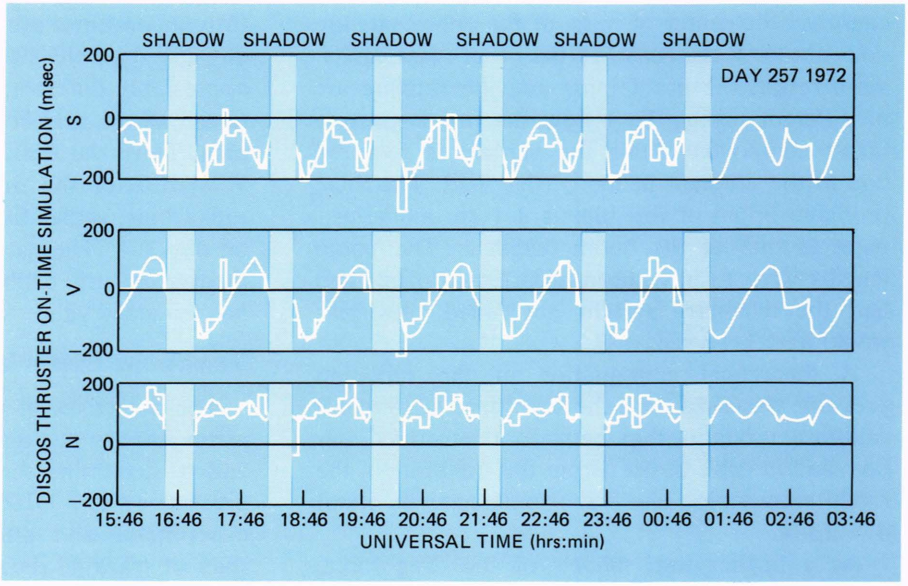

Fig. 3—Thruster on-time data, day 257.

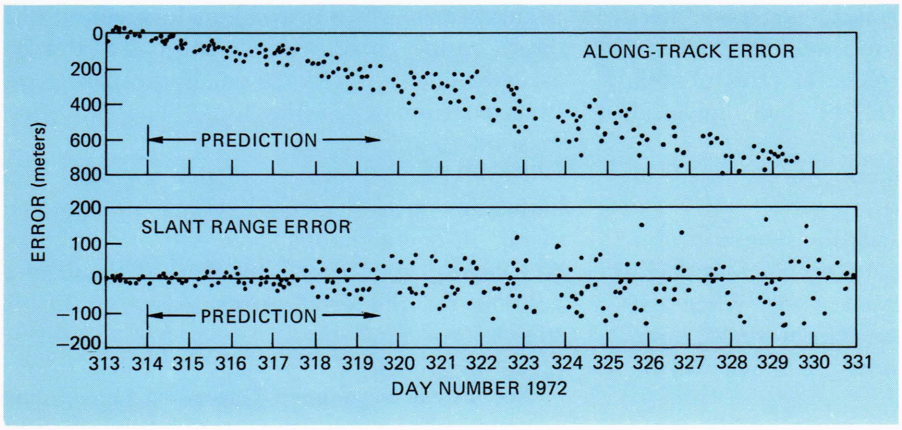

Fig. 4—Results for 14-day prediction.

squares) fit is performed on a set of six orbital constants that determine the motion (e.g., a set of initial conditions for numerical integration). After tracking we are left with the best possible fit of the theoretical satellite motion to the data taken over the tracking span. This orbit is then predicted into the future, and the accuracy of the predicted ephemeris can be measured by "station navigation."

The station navigation process partitions the ephemeris errors for a single doppler pass by simultaneously fitting a station position and a mean satellite oscillator frequency to the doppler residuals. The doppler data residuals are obtained by computing the theoretical doppler shift using the predicted ephemeris and the known station position. The movement of the station location in this fit compensates for ephemeris errors and resolves the errors into the "along-track" and the "slant-range" directions. (The slant-range direction is determined by the vector from station to satellite at time-of-closest approach. The along-track direction is normal to the slant-range in the direction of satellite motion.)

Station navigation provides a direct measure of ephemeris errors that are slowly varying over the time of the pass; the less important, high frequency errors (greater than orbital frequency) tend to be averaged out by the navigation fit. The navigation results are convenient to work with since they condense much of the information contained in an entire doppler pass into two meaningful parameters—the along-track and slant-range errors.

Unmodeled or incorrectly modeled forces can be related to the along-track and slant-range errors by appropriate orbit perturbation techniques. The algebra is fairly complicated, so only the important qualitative behavior will be mentioned here. In particular, the main effect of an unmodeled along-track force bias is a quadratic growth of the along-track error $\delta\ell$, according to the equation:

$$\delta\ell = -\frac{3}{2}F_\ell t^2$$

where F_ℓ is the along-track force bias. For a self-bias force of 10^{-11}g, the growth would be about 110 meters in ten days. In thirty days the error would grow to about one kilometer.

The results for one of the early two-week prediction spans is shown in Fig. 4. Each point in the figure represents the navigation results for a single 15-minute doppler pass at the TRANET site. The predicted ephemeris was based on a two-day tracking span for days 312–313, 1972. The gravity model used for the theoretical satellite motion was the APL 5.0–1967 model.[3]

By present standards, the results in Fig. 4 are not very good. Some degradation in the tracking accuracy was expected with TRIAD since it is in a distinctly lower than usual orbit (750 km altitude). At a lower altitude, the higher-order gravity effects are felt more strongly, and the satellite motion is thus more sensitive to errors in the earth gravity model.

To improve these results and to obtain a reliable estimate of the DISCOS self-bias force, we had to solve four different problems:

1. Correct errors in the earth-gravity zonal harmonics.

---

[3] S. M. Yionoulis, F. T. Heuring, and W. H. Guier, "A Geopotential Model Determined from Satellite Doppler Data at Seven Inclinations," *J. Geophys. Res.* **77**, No. 20, Jul. 10, 1972, 3671–3677.

2. Correct certain resonance, non-zonal harmonics.
3. Correct for the difference in rates of universal solar time (UT1) and universal atomic time (UTC).
4. Change the integration process used to generate the ephemeris.

These four problems are hardly significant for one-day orbit predictions, and were not considered a great deal in previous orbit work. When the prediction times become measured in weeks, however, they become quite important. Each will be discussed in turn.

Zonal Harmonic Effects—The description of the earth's external gravity field used in satellite tracking is an expansion in terms of spherical harmonics. The zonal harmonic terms are those that have no longitude dependence; the non-zonal terms depend on earth longitude. (The second-degree zonal which describes the earth's oblateness is the largest perturbing force for near-earth satellites.) For predicting satellite orbits over spans longer than a day or so, the zonal harmonics are more important than the non-zonals. The reason is as follows.

Because of their longitude dependence and the rotation of the earth under the orbit, the non-zonal harmonics tend to cause periodic orbit perturbations with periods that are multiples of the earth's rotational period (an exception to this is certain "resonance" harmonics which will be discussed in the next section). The effect of the zonal harmonics, however, is unmodulated by the earth's spin, and the resulting perturbations include terms that are linear in time (secular) and long periodic (e.g., 120 days). Because of their low frequency structure, the perturbations from the zonals can grow to sizable amplitudes. They become one of the major effects on satellite motion over arcs longer than a few days. A small error in the zonal coefficients causes a surprisingly large satellite position error in long arc predictions.

In along-track and slant-range, the effect of an error in the zonal coefficients causes an orbital frequency error having a secular and long period envelope. Much of the growing type "scatter" in Fig. 4 is coming from this effect. Orbit frequency errors appear as scatter in the figure since the orbital frequency (14 revolutions/day) is faster than the sampling rate. There is also a contribution to the along-track error, due to the odd degree zonals, which resembles a long period (120 days) cosine curve. Over a short arc, this can easily be confused with the quadratic error growth expected from a self-bias force. The two effects must be carefully separated.

The APL 5.0 gravity model contains zonal harmonics up to degree 12. Using the data span in Fig. 4, one additional even and one additional odd degree zonal coefficient were determined to account for the orbit errors arising from this source.

Non-Zonal Resonance Terms—When an earth satellite makes nearly an integral number of orbits per sidereal day (one revolution of the earth in inertial space), it becomes resonant with certain non-zonal harmonic gravity terms. The resonance phenomena occur when the periodicities in the gravity force arising from a particular harmonic term become commensurate with the orbital period. The satellite response to this resonance is the classical behavior of a forced mechanical system. Orbital perturbations are produced which have a large amplitude and long period; the nearer to resonance the orbit is, the larger the amplitude and longer the period of the resulting perturbations.

The principal resonance terms are those whose order is equal to the nearest integral number of revolutions the satellite makes per sidereal day. For the altitude of the navigation satellites these terms are the 13th- and 14th-order terms. (For synchronous satellites they are the first-order terms.) Even though the effects of the higher order harmonic terms are generally fairly small, near resonance they can be quite large. For this reason, resonant orbits are purposely avoided in most cases.

With a period of 14.3 revolutions per day, the TRIAD orbit is not strongly resonant. The principal resonance term is of 14th order, with a characteristic period of 3.82 days; a weaker resonance with the 15th-order non-zonal harmonics has a period of 1.35 days.

From the data shown in Fig. 4, an along-track, 3.82-day resonance error of 22-meter amplitude was corrected with an odd degree, 14th-order non-zonal. A smaller, 10-meter amplitude term was corrected with an odd degree, 15th-order non-zonal. At the present time, with the other errors removed, there appears to be a relatively small, 2.5-day resonance effect coming from the 28thorder harmonics, although we have not yet tried to determine it.

These errors are not very large. However, they were making an important contribution to the errors shown in Fig. 4. In tracking, the satellite period is controlled by the along-track position error. A low frequency along-track error, such as caused by the 14th-order resonance term, causes the period to be poorly determined in a one- or two-day track. Thus, the small error in the resonant, non-zonal coefficients was causing a sizable error in determining the satellite period. A period error results in a linear growth of the along-track error with time. It is thus important to obtain a precise estimate of the period when the orbit is to be extrapolated far into the future.

Correction for Variations in Universal Time (UT1)—Universal, or mean solar time, is the basis for most civil timekeeping, and it is set by the rotation of the earth and the motion of the sun. Because of both predictable and unpredictable variations in the earth's rotation rate, it is not a uniform measure of time and contains small fluctuations as well as small secular changes in rate. The universal time that represents the true angular rotation of the earth is called UT1, and it can be determined only by direct observation of the sun or stars. In this time measure, the occurrence of midnight and the length of a day are determined by the astronomically observed meridian crossings of the sun and stars.

For a long time, before the variability in the earth's rotation rate was recognized, UT1 was the precise measure of time used for astronomical work. Since the advent of atomic clocks, a more uniform measure of time has been available: atomic time. One such time base, called UTC, is maintained by the U.S. Naval Observatory to be a close approximation to universal time. UTC runs at a fixed rate set by atomic clocks, and the epoch is maintained within one second of UT1 by semiannual "leap seconds" as required.

The time base used for the doppler data collection and the orbit integration is UTC. An orbit can be predicted as a function of UTC in inertial space; however, UT1 is required to determine the correct station position in inertial space at the time of the pass. Thus, the difference UT1 − UTC must be known to accurately navigate with the predicted ephemeris.

Another statement of the problem may clarify this. To predict an ephemeris that is useful for earth navigation, two steps are required. First, the path of the satellite in inertial space must be computed. Then the orientation of the earth *under* the orbit must be known. Any error in this orientation causes a longitude error in navigation.

Over a short span (several days), the difference between UTC and UT1 is nearly constant and can be ignored, the net effect being the generation of a bias in the orientation of the orbit plane when the satellite is tracked. Over long spans, the drift in the difference UT1 − UTC causes an apparent drift in the orientation which cannot be superficially removed in tracking. During the time span in Fig. 4, the difference UT1 − UTC changed by about 0.07 sec, causing an apparent shift in the node of 50 meters. The astronomically-observed values of UT1 were used to correct for this effect in the TRIAD experiment.

Orbit Integration Errors—The results shown in Fig. 4 are based on a satellite ephemeris generated by numerical integration in cartesian space. The method used is a double precision, 8th order Cowell integration using the Adams-Moulton multistep predictor-corrector technique and a Runge-Kutta start-up. This could justifiably be called a "Cadillac" numerical integrator. For computational efficiency, we have also developed an analytic, seminumerical ephemeris generator for use in long span predictions and data analysis. This (hybrid) method computes the short period perturbations (those depending on the phase of the satellite in orbit) by standard second-order analytic techniques and numerically integrates the long period and secular effects with a time step of one satellite revolution. The main advantage of this technique over numerical integration is its speed in extrapolating an orbit for long time periods. Numerical orbit integration, though more precise, is very costly in computing.

Although we had considerably more confidence in the precision of the numerical integrator for short spans, we were surprised to find that the analytic method did considerably better over the 17-day span in Fig. 4. (The differences in the peak errors were nearly 50 meters.) By all measures we have made, the numerical errors with an integration step of 60 seconds should be no more than a few meters in 17 days. Tests on the numerical integrator have included: integrating the pure central earth problem for which the solution is known in closed form, testing the known (Jacobi) energy integral for the full gravity field, closure, and self-consistency with different time steps.

The subject is still controversial, and we have tentatively concluded that there may be an error in computing one of the small forces in the numerical integrator. The effects that are modeled in the theoretical satellite motion include:

1. Higher order earth gravity harmonics.
2. Direct sun and moon gravity effects.
3. Solar and lunar earth body tides.
4. Atmospheric drag.
5. Solar radiation pressure.
6. Precession and nutation of the earth's spin axis.

With all of these effects included, the orbit integrators are very large programs that are difficult to debug. One of the problems is that there is no absolute measure of their errors since a closed form solution does not exist. They can be tested against real life data as in the TRIAD experiments, but then it is difficult to separate numerical errors from errors in the force model or in the initial conditions. The only other measure is to compare one double precision integrator to another; in a sense, the problem is much like the problem of "testing" a precision atom oscillator.

Figure 5 shows the prediction errors over the same span as Fig. 4 *after* the above corrections were made. The ephemeris used in Fig. 5 was generated by the analytic technique rather than numerical integration. Also, the tracking span to start the prediction was increased to three days (313–316) to improve the orbit period determination.

The results are much improved. The errors after a two-week prediction are still on the order of 100 meters, and there is no evidence of an along-track self-bias force as large as 10^{-11}g. However, there are still "growing" type errors that we have not completely eliminated.

The next step was to increase the prediction span to 40 days (and 40 nights). With this length span, it is more convenient to test the predicted ephemeris at intervals, rather than continuously, along the span. Accordingly, the navigation tests were made with a day's worth of passes (about twenty) every fifth day. Figure 6 shows the resulting along-track error. The ephemeris was predicted from a six-day track (days 320–326). The plotted points are the average along-track errors for the passes navigated over that day. The bars show the limits on the RMS "scatter" for the passes that day. The slant-range results are not plotted, but they had about the same magnitude scatter.

There is clearly a quadratic along-track error growth in Fig. 6. This growth is consistent with a constant along-track self-bias force of about 5×10^{-12}g. To test this, a constant along-track force of this magnitude was modeled in the orbit integration program, and the test was repeated with a still longer prediction span. These results are shown in Fig. 7. Again, each point plotted is based on the average navigation results of about twenty passes.

Although the quadratic effect has been removed, there are still structured, secular errors in the predicted orbit. The origin of at least a portion of this secular error is understood. Some error is due to an orbit period error, and some is due to residual errors in the zonal harmonics.

The period error after tracking is determined by the precision in the data and force model, and by the length of the span. The tracking precision over the six-day span, as measured by the RMS residual errors, is about 25 meters in satellite position. With this level of precision, a period

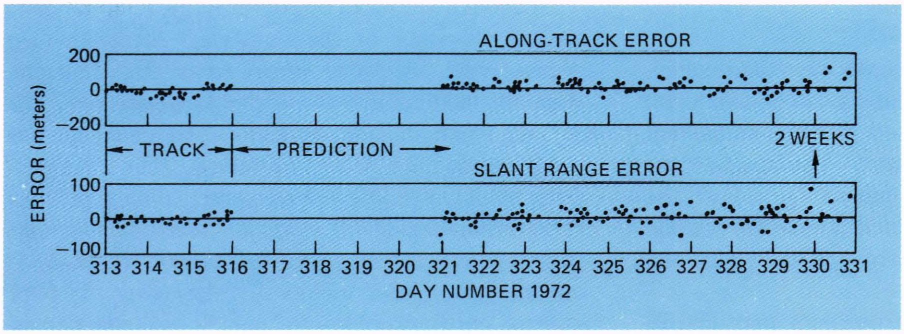

Fig. 5—Results for 14-day prediction, after corrections.

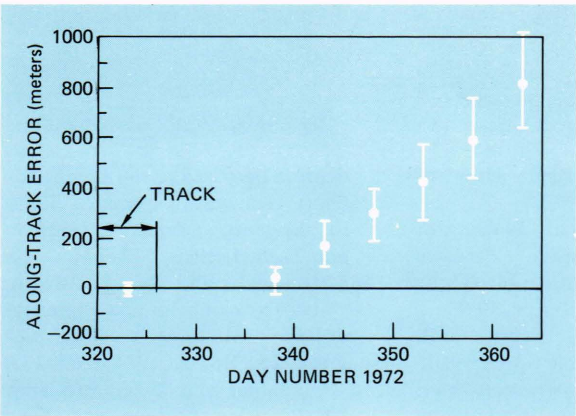

Fig. 6—Along-track orbit errors.

error equivalent to at least one meter a day along-track growth is reasonable to expect. Thus, in 60 days the effect could easily grow to 60 meters or more. In general, this error can be reduced by using a longer tracking span to start the prediction.

The effect of the zonal harmonics has already been discussed. For spans as long as 60 days, the orbit errors are extremely sensitive to the errors in the zonal coefficients. The corrections to the zonals that were determined from the 14-day span were obviously not precise enough to keep the errors under 100 meters for 60 days. Correcting the zonals to this level requires a careful analysis of a span at least 120 days long, since this is the period of the principal perturbation coming from the odd zonals.

Concluding Remarks

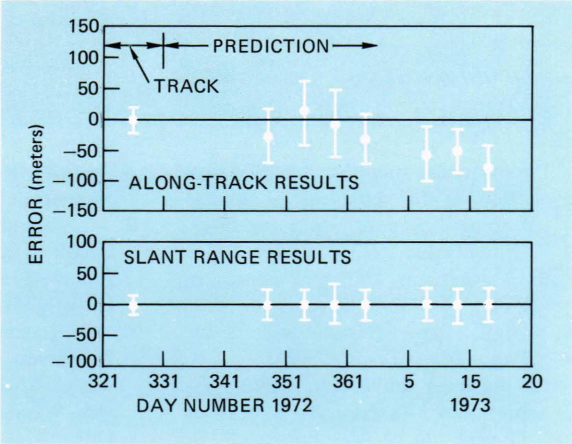

Fig. 7—Orbit errors for 60-day prediction.

One thing should be apparent from the description of these tracking experiments: Once the drag and radiation pressure effects were removed from the orbit, new problems emerged that up to now had been masked, mainly by the drag error. This was one of the exciting things about working with the TRIAD tracking data. It is a good example of a common pattern in science. The new problems have not been completely solved. But it is clear that with a DISCOS device we can realistically talk about accurate orbit predictions of months instead of days for navigation satellites.

The experiments showed conclusively that the along-track self-bias force was less than 10^{-11}g for the first several months of the satellite's life. However, we still have not checked the long-term reliability of the system. There is no reason to expect that the self-bias force would change after being stable for several months. However, it probably should be checked at a later point in its lifetime to ensure that the system is still properly operating.

Acknowledgment

The analysis of the thruster on-time data was done by A. Eisner and R. Yuhasz (now at Syracuse University).