

# TROPOSPHERIC REFRACTION EFFECTS ON SATELLITE RANGE MEASUREMENTS

Helen S. Hopfield

*Radio or optical signals traveling between an earth station and an object in space must pass through the earth's atmosphere, where the signal velocity is not the same as in free space. This altered velocity introduces an error into measurements of range or range-rate. The signal velocity depends on local atmospheric conditions, hence its perturbing effect on range measurement is a function of time and place, as well as elevation angle of the signal path. An atmospheric model is described here for predicting the magnitude of the effect on the basis of local meteorological conditions at the earth-based tracking station.*

## Introduction

**T**O DETERMINE THE ORBIT OF AN EARTH SATELLITE, or conversely, to use a known satellite orbit to find the location of a point on the earth, it is necessary to use some sort of electromagnetic signal (light or radio signal) passing between the satellite and the earth station. This signal traverses the earth's atmosphere, where its velocity is not the same as in free space (the index of refraction of the atmosphere is not unity). Thus the signal travel time is slightly altered by the presence of the atmosphere. This means that there is an error in the measured range; there is also an error in the range rate, since the amount of the range error varies. Even if the atmosphere is in a steady state, its effect is greater for an oblique than for a vertical signal path. In addition, the amount changes with weather, since it depends on the integrated effect of air pressure, temperature, and water vapor along the path.

These effects are large enough to be significant when precise work is to be done with satellites, and an atmospheric correction is therefore needed. To make this correction, two parts of the atmosphere are so different that they must be con-

sidered separately: the *lower, un-ionized part*, which is a few tens of kilometers in height (troposphere and stratosphere, which can be lumped together); and above that, the *ionized part* (the ionosphere), which extends upward for several hundred kilometers.

These two parts affect signal velocity quite differently. The *ionosphere* has negligible effect on visible light, but a significant, and frequency-dependent, effect on a radio signal. Thus if the same range or doppler shift is measured simultaneously at two different radio frequencies, the ionospheric effect, to a good approximation, can be cancelled from the data. This two-frequency method is used in the navigational satellite system, and corrects for first-order ionospheric effects.<sup>1</sup>

The *un-ionized troposphere and stratosphere*, on the other hand, affect both visible light and radio signals, but here the index of refraction does not vary with frequency in the radio range (at least up to frequencies of 15 GHz). Thus the two-

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<sup>1</sup> W. H. Guier and G. C. Weiffenbach, "A Satellite Doppler Navigation System," *Proc. IRE* 48, April 1960, 507-516.



frequency method that corrects radio measurements for the ionosphere does nothing to remove lower atmosphere effects, and another method is needed.

These lower-atmosphere effects and the model that has been developed at APL to correct for them are the subject of this paper. For brevity, this tropospheric-and-stratospheric effect will generally be called the "tropospheric effect;" at least 80% of it actually occurs in the troposphere, below the tropopause.

## Preliminary Theory

When a signal passes through an inhomogeneous medium such as air, where its velocity changes from point to point, not only is its travel time affected, but the path of the signal is also bent. The relative importance of the change in travel time versus the change in direction depends on what is being measured. When the basic measurement is the satellite range or its rate of change (e.g., the doppler shift of the radio signal), the important atmospheric effect is the change in signal travel time; i.e., the velocity effect itself, and not the angle. Here, bending of the signal path is not important unless it is large enough to affect the actual signal path length significantly. This occurs only at elevation angles so low that they are generally not used in practice. At least for a preliminary study, path curvature can be neglected. The requirement is to find an atmosphere model that will properly account for the total retardation of the transmitted signal at different locations, seasons, and elevation angles. This total retardation is equivalent to the atmospheric error in the measured range.

If the index of refraction were known all along the signal path whenever a range measurement was to be made, the refraction error in the path could easily be computed. In practice, however, such detailed information is not available. A mathematical model is therefore needed: one that will give a sufficiently accurate estimate of the whole refractive effect on range at any location, on the basis of observed (or nominal) surface conditions; the estimate must be adequate at all elevation angles that are to be used.

To simplify the problem, it will be assumed that the atmosphere is horizontally stratified in the region traversed by the signal (i.e., only vertical gradients are present; no horizontal gradients of

pressure, temperature, and water vapor content); that signal path bending is small enough to be negligible; and that conditions do not change significantly during the interval of observations (e.g., a satellite pass). Weather fronts are disregarded for the time being.

The amount of the tropospheric range error  $\Delta s$  in a signal passing through the troposphere at any angle is given by the expression

$$\Delta s = \int (n - 1) ds, \quad (1)$$

where  $n$  is the index of refraction of air (varying along the path) and the integral is taken along the path, here assumed to be a straight line. In order to evaluate this, it is convenient to define a quantity called the refractivity  $N$ , as

$$N = 10^6 (n - 1). \quad (2)$$

This refractivity  $N$ , for air, can be expressed as the sum of two parts,  $N_d$  and  $N_w$ , the so-called "dry" and "wet" components. Expressions for these are known:<sup>2</sup>

$$\left. \begin{aligned} N_d &= 77.6 \frac{P}{T} \\ N_w &= 3.73 \times 10^5 \frac{e}{T^2} \\ N &= N_d + N_w \end{aligned} \right\} \quad (3)$$

where  $T$  is the absolute (Kelvin) temperature of the air,  $P$  is its pressure in millibars, and  $e$  the water vapor pressure, also in millibars.

Both  $N_d$  and  $N_w$  decrease with height above the surface of the earth, but at different rates. (Only a little water vapor is found above a height of 6 km, but about half of the air is above that height.)

A meteorological balloon is instrumented to measure pressure, temperature, and water vapor content of the atmosphere as the balloon rises. Such data can be used in Eqs. (3) to compute the actual height profile of  $N$ , and of its components, at the time of the balloon ascent. Numerical integration of  $N$  with height then yields the value of the tropospheric range error in a signal arriving vertically; e.g., a height error  $\Delta h$  in the measured height of an overhead satellite.<sup>3, 4, 5</sup>

<sup>2</sup> E. K. Smith, Jr. and Stanley Weintraub, "The Constants in the Equation for Atmospheric Refractive Index at Radio Frequencies," *Proc. IRE* **41**, Aug. 1953, 1035-1037.

<sup>3</sup> H. S. Hopfield, "Two-Quartic Tropospheric Refractivity Profile for Correcting Satellite Data," *J. Geophys. Res.* **74**, Aug. 20, 1969, 4487-4499.

<sup>4</sup> H. S. Hopfield, "Tropospheric Effect on Electromagnetically Measured Range: Prediction from Surface Weather Data," *Radio Sci.* **6**, Mar. 1971, 357-367.

<sup>5</sup> H. S. Hopfield, *Tropospheric Range Error Parameters: Further Studies*, APL/JHU Report CP 015, June 1972; also NASA Report X551-72-285, Aug. 1972.



If we combine Eqs. (1) and (2), writing the components separately, we have

$$\left. \begin{aligned} \Delta h_d &= 10^{-6} \int N_d dh \\ \Delta h_w &= 10^{-6} \int N_w dh \\ \Delta h &= \Delta h_d + \Delta h_w \end{aligned} \right\} \quad (4)$$

where the integration is carried through the troposphere.

### Studies with Meteorological Balloon Data

One weather balloon would enable us to evaluate  $\Delta h$  at one time and place, but in view of global differences of climate and local weather variations, it is necessary to examine values of  $\Delta h$  observed at different times and places. In the study described here, these values have been numerically evaluated from the meteorological data provided by several thousand balloon ascents made at widely separated locations in both northern and southern hemispheres. The data were obtained from the U. S. National Climatic Center (NOAA) in one-year sets (two balloons per day) from each location ranging from Point Barrow, Alaska to Byrd Station, Antarctica. Each balloon ascent provides data on pressure, temperature, and water vapor pressures at a set of heights (usually 50 or 60 observed points per balloon ascent). Profiles of  $N_d$  and  $N_w$  and their height integrals (the height error components) were obtained from the above equations for each balloon flight separately. The computer was programmed to delete occasional flights that did not go high enough to provide full data.

Figures 1 and 2 show *surface* refractivity as a function of time during a year at Washington, D. C. (Dulles Airport) and at Pago Pago, Samoa, respectively. The wet and dry components of the radio refractivity at the surface are shown in each figure. In Washington (Fig. 1) both components show seasonal variations (opposite in phase); the amplitude of the  $N_w$  variation overshadows the  $N_d$  amplitude, so that the total  $N$  is greater in summer. At the surface,  $N_w$  is often as much as 30% of the total  $N$  in summer, though much less than that in winter. Both components show marked weather effects. When the weather is stable (as in Fig. 1, summer) there is a clear diurnal variation of  $N_d$  reflecting the diurnal temperature variation (cf. Eq. (3); pressure has little diurnal change).

Seasonal variations of the surface  $N$  are much

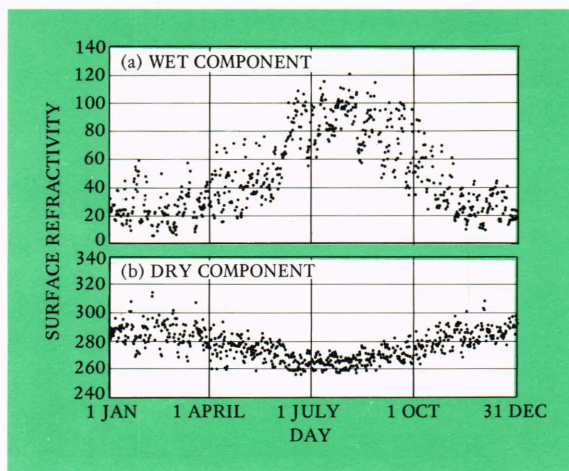


Fig. 1—Refractivity at the surface during a year, Washington, D. C., 1967.

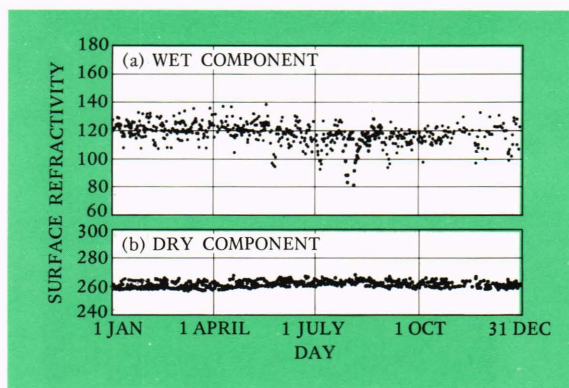


Fig. 2—Refractivity at the surface during a year, Pago Pago, Samoa, 1967.

smaller in tropical Samoa (Fig. 2) and pertain to southern hemisphere seasons. In Samoa,  $N_w$  is always a large fraction of the total  $N$ . Diurnal effects are present, but are small, in  $N_d$  (diurnal temperature changes are small). Weather effects are smaller than in Washington.

Figures 3 and 4 show *height integrals* of  $N$  (i.e.,  $\Delta h$ ) and its components, for the same two locations, for comparison with Figs. 1 and 2, respectively.

In the dry component especially, the height integral does not follow the annual cycle of the surface value of  $N_d$ . In fact, the height integral of  $N_d$  is nearly constant throughout the year at each station; and further, it is practically the same for Washington and Samoa (2.3 meters), in spite of climatic differences. But the surface values of  $N_d$  at the two places are different.



Also, the wet component integral is seldom more than 10% of the total integral, whereas at the surface,  $N_w$  was sometimes as much as 30% of  $N$ .

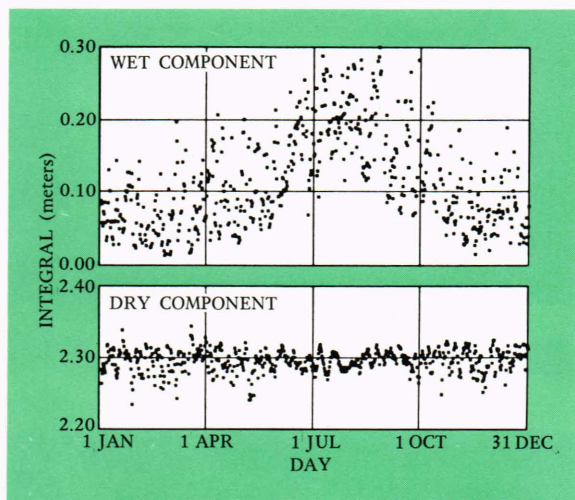


Fig. 3—Vertical integrals of refractivity during a year; balloon data, Washington, D. C., 1967.

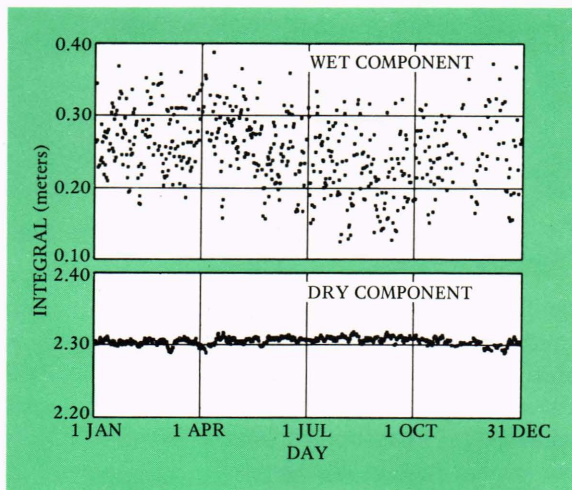


Fig. 4—Vertical integrals of refractivity during a year; balloon data, Pago Pago, Samoa, 1967.

### Relation of Vertical Range Error to Surface Pressure

The dry component of  $N$  will be examined first. Clearly, the height integral of  $N_d$  is not proportional to its surface value. This can be explained on the basis of the gas laws and the hydrostatic equation, for a dry atmosphere that is in equilibrium.

If  $V$  is the volume and  $R$  is the gas constant for unit mass of air, we have, from the gas laws,

$$PV = RT.$$

Then

$$\frac{P}{T} = \frac{R}{V} = R\rho, \quad (5)$$

where  $\rho$  is the density. If this is combined with the dry part of Eq. (3), the result is

$$N_d = 10^{-3} \times 77.6 R\rho$$

if  $R$  and  $\rho$  are in cgs units. Then the height integral becomes

$$\int N_d dh = 10^{-3} \times 77.6 R \int \rho dh. \quad (6)$$

But from the hydrostatic equation, the pressure at the base of a column of air (i.e., at the earth's surface) is

$$P_s = \int g\rho dh \quad (7)$$

where  $g$  is the acceleration of gravity. Combining Eqs. (4), (6) and (7), and assuming that the variation of  $g$  in the troposphere is negligible, we get, for the zenith range effect in dry air,

$$\Delta h_d = 10^{-6} \int N_d dh = 10^{-9} \times \frac{77.6 R P_s}{g} \quad (8)$$

if all quantities are in cgs units.<sup>4</sup>

Thus the height measurement is theoretically independent of temperature and the  $N_d$  profile, and is a linear function of surface pressure only, assuming that  $g$  and  $R$  are constant.

This theoretical result can be checked by means of balloon data, dealing now with the dry component of the real atmosphere. Figure 5 shows samples of  $\Delta h_d$  from balloon data, plotted against the surface pressure  $P_s$  at the time of the balloon flight, for two locations: Washington, D. C. (data from one month) and Byrd Station, Antarctica (data from two months). In both cases, the plotted points lie along a straight line, with little scatter. The slopes of the lines are very nearly the same. That is, the proportionality constant relating  $\Delta h_d$  to  $P_s$  is nearly the same at the two places in spite of the fact that Byrd Station is several tens of degrees colder and almost 1500 meters higher above sea level than Washington.

Equation (8) may be written

$$\Delta h_d = k P_s, \quad (9)$$

where theoretically

$$k = 10^{-9} \times 77.6 \frac{R}{g} \quad (10)$$

and  $k$  is a constant. But  $k$  may also be determined



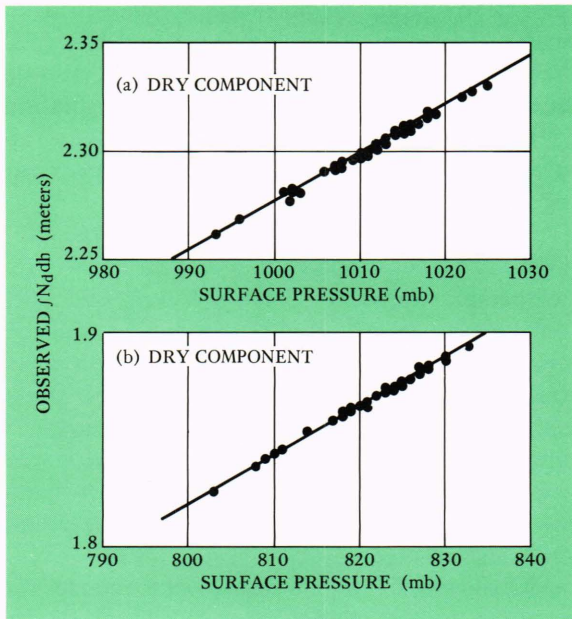


Fig. 5—Vertical integrals of dry component of refractivity versus surface pressure. (a) Washington, D.C., March 1967, (b) Byrd Station, Antarctica, January and July 1967.

empirically from observations and Eq. (9). This was done for each one-year set of balloon data. These  $k$  values, with surface pressure and Eq. (9), yield computed values of  $\Delta h_d$  which agree with the observed values within 2 mm or less (RMS) at each station; i.e., the scatter is less than 0.1%. (Incidentally, the small scatter implies that departures from atmospheric equilibrium were small.)

The  $k$  values are nearly the same for all stations, but not precisely: they exhibit a latitude variation of 0.8% from equator to pole. The latitude variation of  $g$  (cf., Eq. (10)) accounts theoretically for more than half of this. The remainder is an indirect effect of water vapor, although we are dealing with the so-called “dry” effect. The value of  $R$  per gram of air (Eq. (10)) is a function of the “molecular weight” of air, which depends on its water vapor content. Equation (10) can be used with the empirical values of  $k$  to derive the annual mean “molecular weight” of the air above each station, and to get from this the mean percent of water vapor molecules present. The results range from 1% of water vapor near the equator to 0.2% at Byrd Station. Great precision cannot be claimed for this method of measuring water vapor content, but the results are of reasonable magnitude.

The theoretical reasoning that led to Eq. (8) cannot be applied to the wet component alone, because of the imperfect mixing of water vapor with the other air molecules. There is as yet no comparable expression for the wet component. The prediction of  $\Delta h_w$  will be discussed later.

At least for the dry component, we have found that the shape of the  $N$  profile is immaterial for predicting the atmospheric effect at or near the zenith. This is not true, however, at low elevation angles, and the profile shape will now be considered.

### **$N$ Profile Model for Use at all Elevation Angles**

A successful model profile must, of course, provide a zenith integral of  $N_d$  that is consistent with Eq. (8); but for low angle use, it must also give a reasonable approximation to observed  $N$  profiles. The model that will be described treats each component of  $N$  (dry and wet) as a different function of its surface value and of height above the earth.

In a dry, isothermal atmosphere, the refractivity  $N$  is theoretically an exponential function of height above the surface (neglecting height variation of  $g$ ). Exponential approximations have been extensively used to represent both the dry and wet component profiles of  $N$ .<sup>6,7</sup>

In the earth's atmosphere, however, the temperature decreases with height at a fairly constant rate in the troposphere (here meaning the troposphere proper), is fairly constant in the tropopause region, and above that increases slowly with height in the stratosphere.

Considering the troposphere proper, let us assume that the lapse rate  $\alpha$  is constant (where  $\alpha = -dT/dh$ ). In this case, pressure is not an exponential function of height,<sup>8</sup> and the  $N_d$  profile is also not exponential,<sup>3</sup> but can be put in the form

$$N_d = N_{ds} \left[ \frac{T_s - h}{\frac{T_s}{\alpha}} \right]^\mu, \quad (11)$$

<sup>6</sup> B. R. Bean and E. J. Dutton, *Radio Meteorology*, National Bureau of Standards Monograph 92.

<sup>7</sup> B. R. Bean, “Concerning the Biexponential Nature of the Tropospheric Radio Refractive Index,” *Beitr. Phys. Atmos.* **34**, Nov. 1/2, 1961, 81-91.

<sup>8</sup> Bernhard Haurwitz, *Dynamic Meteorology*, McGraw Hill Book Co., New York, 1941.



where

$$\mu = \frac{g}{R\alpha} - 1. \quad (12)$$

The quantity  $T_s/\alpha$  has the dimensions of height, and may be considered a height parameter of the profile. The degree of the  $N_d$  profile depends on  $\alpha$ . A lapse rate of  $34^\circ\text{C}/\text{km}$  would cause the (dry) atmosphere to be a "slab" of air of constant density, if  $g = 980 \text{ cm}/\text{s}^2$ . A lapse rate half as large ( $17^\circ\text{C}/\text{km}$ ) would result in a linear decrease of air density, and refractivity, with height. But these are improbably high values of  $\alpha$ . The adiabatic lapse rate is  $9.8^\circ\text{C}/\text{km}$ ,<sup>8</sup> but observed tropospheric values are generally less ( $\sim 7^\circ\text{C}/\text{km}$  in warm climates, less near the poles);  $\alpha$  and  $\mu$  are negative in the region of a temperature inversion, and negative in the stratosphere.

The theoretical zenith integral of  $N_d$  in an atmosphere with constant lapse rate, whether  $\alpha$  is zero (exponential  $N_d$  profile), positive, or negative (Eq. (11)), is identical to its value in Eq. (8). If the actual lapse rate is constant and is known, it is possible to write an equation for the  $N_d$  profile that will match the observed profile as regards both zenith integral and profile shape. Since, in practice, the lapse rate is only fairly constant in the troposphere and changes sign at the tropo-

pause, any single mathematical function will not match the actual  $N_d$  profile shape perfectly at all heights. A single function can, however, yield the correct zenith integral regardless of irregularities in the profile shape and *also* provide a usable approximation to the profile shape in the denser part of the atmosphere.

A fourth-degree model<sup>3</sup> ( $\mu = 4$  in Eqs. (11) and (12)) corresponds, if  $g = 980 \text{ cm}/\text{s}^2$ , to a temperature lapse rate of  $6.8^\circ\text{C}/\text{km}$ , and does, on the average, match observed profiles well to a considerable height in regions of the earth where this lapse rate is a realistic value in the troposphere. Figure 6 illustrates this for a profile that is an average for a month. Height parameters for such a model will be discussed below. Instantaneous profiles, of course, show irregular deviations from the average.

As shown in Fig. 6, a fourth-degree model was used for the wet component as well as for the dry. On the average, it fits the observed data reasonably well although without the same theoretical justification as for the dry part. Height parameters are, however, very different for the two components.

The profile expression for either component of the refractivity is now written in the form of Eq. (11), using a height parameter  $h_d$  and an analogous one  $h_w$ :

$$\left. \begin{aligned} N_d &= \frac{N_{ds}}{h_d^4} (h_d - h)^4 & h \leq h_d \\ N_w &= \frac{N_{ws}}{h_w^4} (h_w - h)^4 & h \leq h_w \end{aligned} \right\} (13)$$

Expressions based on this  $N$  profile can be used to correct range or doppler data at different elevation angles, point by point throughout a satellite pass.<sup>3</sup> Some illustrations of its use will be shown later. An algorithm that simplifies the computation of the correction was developed by Yionoulis.<sup>9</sup> The equivalent heights  $h_d$  and  $h_w$  for the profile must yield height integrals that match observed data, and therefore their values are obtained from balloon data. The height integrals of Eq. (13) are:

$$\left. \begin{aligned} \int_0^{h_d} N_d dh &= \frac{1}{5} N_{ds} h_d \\ \int_0^{h_w} N_w dh &= \frac{1}{5} N_{ws} h_w \end{aligned} \right\} (14)$$

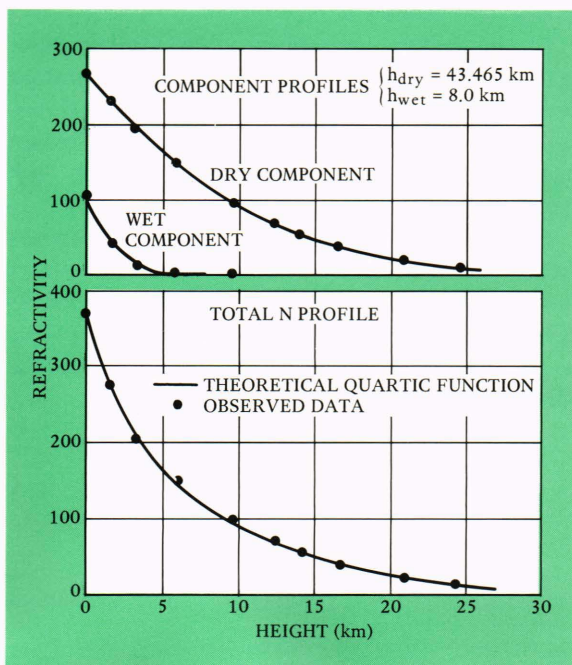


Fig. 6—Profile of tropospheric refractivity  $N$ , weather ship E,  $35^\circ \text{N}$   $48^\circ \text{W}$ , July 1967.

<sup>9</sup> S. M. Yionoulis, "Algorithm to Compute Tropospheric Refraction Effects on Range Measurements," *J. Geophys. Res.* **75**, Dec. 20, 1970, 7636-7637.



TABLE 1  
PARAMETERS FOR TWO-QUARTIC N PROFILE

Station	Year	$h_{d_0}$ (km) Above Station	$a_d$ (km/°C)	Prediction Error for $\int N_d dh$ , $\sigma$ (meters)	$h_w$ (km) Above Station	Prediction Error for $\int N_w dh$ , $\sigma$ (meters)
Weather Ship E	1963	40.028	0.15307	0.001372	11.323	0.028123
Weather Ship E	1965	40.072	0.15087	0.001423	10.705	0.027883
Weather Ship E	1967	40.067	0.15092	0.001509	9.698	0.032824
Ascension Island	1967	40.257	0.14991	0.001127	9.670	0.021204
Caribou, Maine	1967	40.084	0.14839	0.001548	11.064	0.027642
Washington, D.C.	1967	40.112	0.14918	0.001608	11.379	0.030072
St. Cloud, Minn.	1967	40.091	0.14795	0.001308	11.537	0.023500
Columbia, Mo.	1967	40.114	0.14878	0.001569	10.965	0.028744
Albuquerque, New Mexico	1967	40.129	0.14797	0.001393	12.814	0.016624
El Paso, Texas	1967	40.165	0.14765	0.001635	13.013	0.025084
Vandenberg AFB, Cal.	1967	40.123	0.14855	0.001475	8.539	0.024808
Pago Pago, Samoa	1967	40.353	0.14409	0.001684	10.674	0.045408
Wake Island	1963	40.099	0.15239	0.001418	10.600	0.033192
Wake Island	1965	40.141	0.15099	0.001531	9.217	0.032717
Wake Island	1967	40.120	0.15173	0.001669	9.482	0.044369
Majuro Island	1967	40.495	0.14034	0.001489	11.265	0.049702
Point Barrow, Alaska	1967	40.022	0.14738	0.001261	11.507	0.017658
Byrd Station, Antarctica	1967	39.993	0.14675	0.001091	14.042	0.005542
Mean		40.136	0.14872			

The total range (height) effect on a vertically traveling signal is the sum of the two components.

Let us examine the dry component first. Comparison of the dry part of Eq. (14) with Eqs. (3) and (8) shows that Eq. (14) can be consistent with (8) only if  $h_d$  varies directly as surface temperature. This is not unreasonable. It is well known that, for example, the 200-millibars level in a given area is consistently higher in local summer than in winter.

It was therefore assumed that

$$h_d = h_{d_0} + a_d T_c, \quad (15)$$

where  $h_{d_0}$  is the value of  $h_d$  when the surface temperature is 0°C, and  $a_d$  is the temperature coefficient of the variation of  $h_d$  with surface temperature.

The value of  $h_d$  from Eq. (15) was used in (14) to get a theoretical expression for the value of  $\int N_d dh$  corresponding to a given surface  $N_{d_s}$  (i.e., at the time of a balloon flight). Theoretical integrals were equated to observed ones, and a least-squares procedure was used to solve for values of  $h_{d_0}$  and  $a_d$  for each one-year set of data. The results are listed in Table 1. These parameters were then used to compute a predicted value of  $\Delta h_d$  corresponding to each observed integral. The difference is the prediction error; its RMS value for each one-year data set is also listed. It

is 1.7 mm or less in each case; i.e., well under 0.1% of the observed integral (2.3 meters for a station near sea level). Thus  $\Delta h_d$  can be predicted from surface data within 1 or 2 mm by two methods: either from surface pressure alone, or from surface refractivity and the quartic model. The latter method has the advantage that it can be used at low elevation angles also.

The empirical values of  $h_{d_0}$  above the station are closely the same for all the stations (40 km). This empirical result is in close agreement with the theoretical value of  $T_0/\alpha$  (cf. Eq. (11)) for a fourth degree  $N_d$  profile, which corresponds to  $\alpha = 6.8^\circ\text{C}/\text{km}$ . These heights should be interpreted as parameters for matching observed integrals, not as indicating any undulation of actual pressure levels above surface undulations.

Theoretically, Eq. (15) should have been written with no constant term; on simple theory,  $h_d$  should be zero at 0°K. Extrapolating from the empirical values of Table 1, the equivalent height of the model falls to zero in the general neighborhood of 0°K. The deviations may be related to second-order effects that have not yet been investigated (e.g., gravity variations, time of day and atmospheric tides, water vapor, etc.).

A height  $h_w$  for the wet component profile was also derived from the balloon data, on the assumption that  $h_w$  for each station is constant dur-



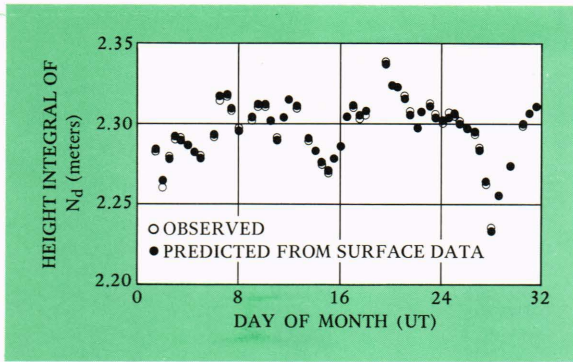


Fig. 7—Observed and predicted vertical integrals of refractivity at Washington, D. C., dry component, January 1967.

ing the year. This assumption may not be valid, but has not been improved as yet. The results, and prediction errors for the model, are also listed in Table 1. There is a good deal of variation in the heights  $h_w$  for different stations, and even for the same station in different years; but  $h_w$  is less than  $h_d$  by approximately a factor of three. The prediction errors for the wet part are measured in centimeters, not millimeters. Predictions for the wet effect are essentially based on statistics, not physical theory, and they are not yet comparable to the dry predictions in accuracy. Much work remains to be done in this field.

Figure 7 shows a one-month sample of observed

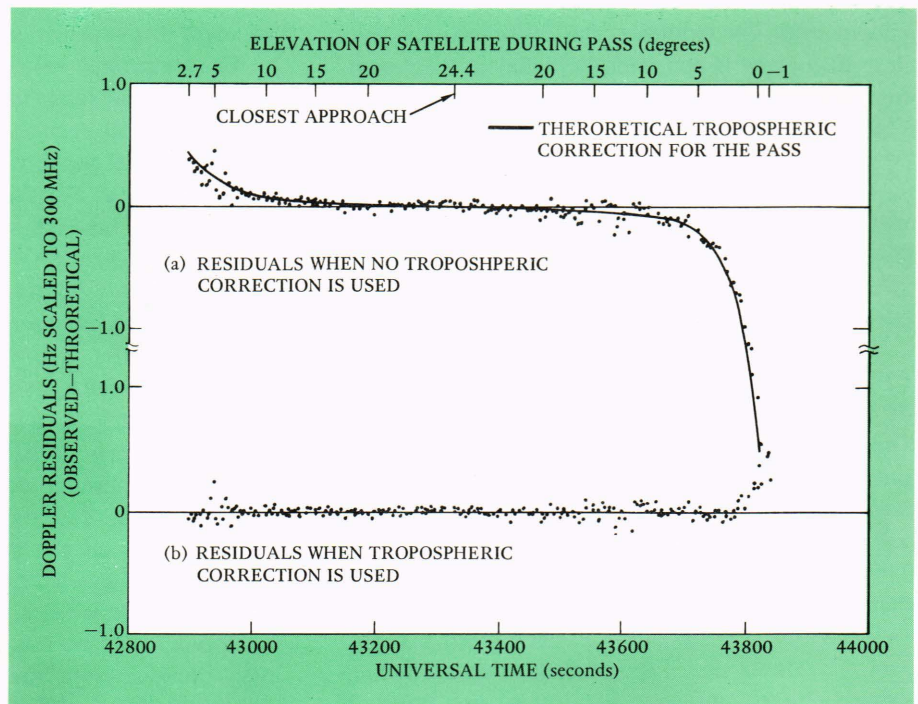
and predicted height integrals (dry component). Each observed point represents data from one balloon. A predicted point is plotted for each observed point, but they cannot always be distinguished on the scale of the figure.

### Use of the Model

It was stated earlier, but without supporting evidence, that the tropospheric effect on satellite data for precise work is large enough to require correction. The following pages will present some data and computed positions *without* and *with* a correction for the troposphere. The corrections pertaining to Figs. 8 and 9 are made on the basis of the two-quartic tropospheric model as described above, but use preliminary values of the height parameters, not the values of Table 1.

At any given point during a satellite pass, the station-to-satellite range appears too great if measured by a timing procedure (e.g., radar). The range effect would obviously be least at the point of closest approach; larger at both ends of the pass. The rate of change of the observed satellite range during a pass therefore appears too large and changes too rapidly; i.e., the slope of the observed doppler curve (essentially range-rate) is too steep. Geometrically (and somewhat paradoxically), this makes the satellite pass appear *closer* to the station than it really is. Figure 8

Fig. 8—Tropospheric effect on doppler data.





shows data for a satellite in a known orbit passing a station of known position (the same pass in both parts).<sup>3</sup> The plotted points are the residual doppler errors during the pass, i.e., the difference between observed and theoretical doppler shift. In the upper graph, no correction was made for the troposphere, and the magnitude of the residual errors increases sharply toward the ends of the pass. The tropospheric correction, computed from the two-quartic model, is shown as the solid curve in the upper graph and is in good agreement with the systematic trend of the residual errors. This computed correction was then applied to the data and the residual errors were re-computed. The new residuals are shown in the lower part of the figure. They now exhibit no appreciable systematic trend, but are centered about zero. (Nontropospheric errors in this particular pass were small.) Although the tropospheric effect is clearly greatest near the ends of the pass, it is important to note that the curve showing the theoretical correction is not horizontal even at the center of the pass. As the data span is reduced to shorter intervals near the center of the pass, the tropospheric effect approaches a limit that is not zero.

The effect of the troposphere on station-to-orbit range is shown in Fig. 9. Navigated position in a direction parallel to the orbit is affected by the troposphere only if the tracking data are not symmetrical about the point of closest approach, and will not be discussed here. The errors in station-to-orbit range are shown here as a function of pass elevation angle at closest approach, for a two-day set of satellite passes observed at tracking stations distributed over the earth. Each point represents one pass.

Part (a) of Fig. 9 shows navigated range errors when no tropospheric correction is used. The uncorrected troposphere has shortened the apparent station-to-orbit range by some 20 meters (average) in the case of high-angle passes, but by much more for lower passes (40 meters for a pass whose maximum elevation is 30° and 100 meters for a 10° pass). These systematic errors were removed by the tropospheric correction (part (b) data used down to 1° elevation). There was, however, a slight overcorrection at very low angles because of the neglected path curvature effect, mentioned earlier. When all data below 5° were deleted, this effect disappeared (part c).

As mentioned above, Figs. 8 and 9 use the de-

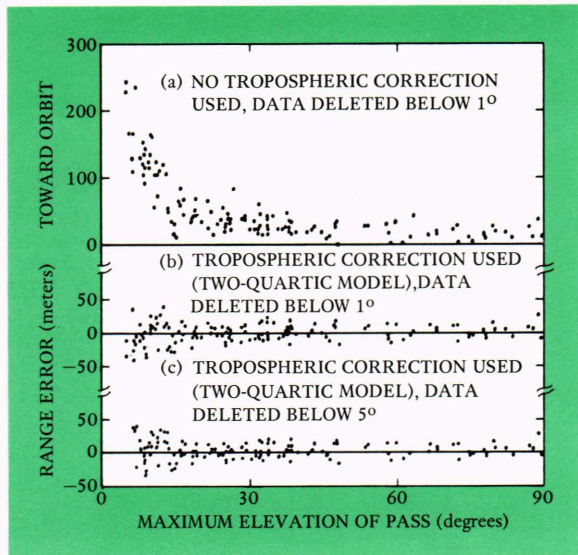


Fig. 9—Navigation errors in station-to-orbit range, satellite 1967 34A, September 1 and 2, 1967.

sired troposphere model but not the most up-to-date parameters for it; use of the new parameters can be expected to help in reducing the average positioning error.

The above discussion has dealt with atmospheric effects on signals that traverse the whole un-ionized part of the atmosphere. It is not directly applicable to low-angle radar measurements of targets within the atmosphere. It can be applied, however, to a variety of satellite radio measurements. The part relating to the dry component, with only minor changes, can also be applied to laser ranging data.

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