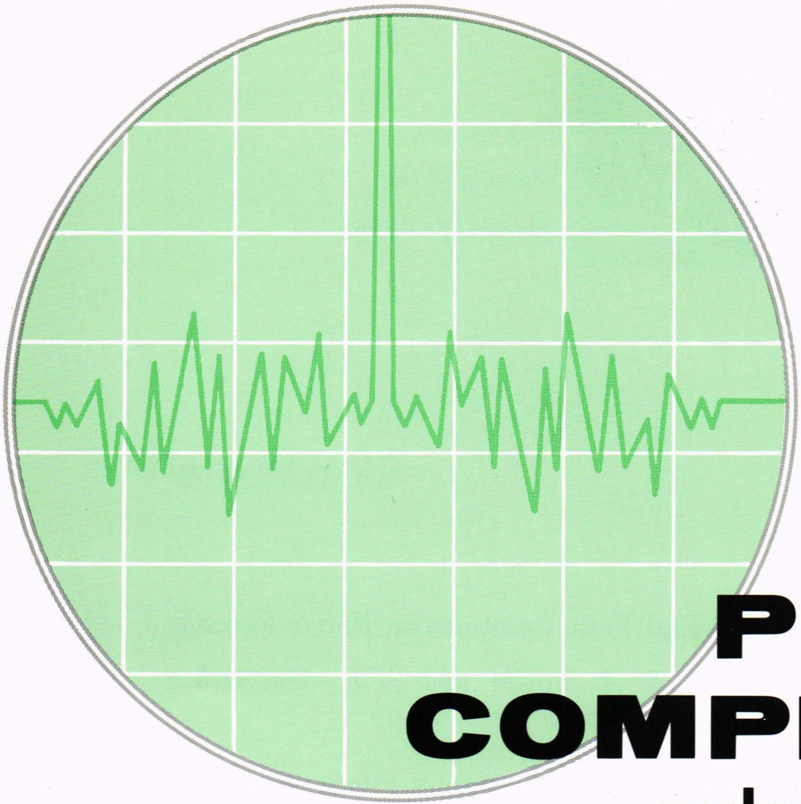

digital PULSE COMPRESSION radar receiver

Modern radar system requirements for both tactical and commercial application are becoming increasingly demanding. Greater detection ranges, finer range resolution, better visibility in clutter, and high reliability are a few of the more pressing goals. Certainly transmitter peak power has not reached a maximum, but order of magnitude increases are hard to attain. Given finite peak power, some of these goals are mutually exclusive unless fairly sophisticated signal management is undertaken.

Pulse compression in one form or another is the only technique which meets the present needs and can be projected as a continuing solution. Fine range resolution requires large signal bandwidth. Signal detectability depends on total energy and is therefore favored by long pulses. Transmission, reception, and correlation of high time-bandwidth product signals constitute "pulse compression."

Typically, transmitter final amplifiers are driven into saturation, so that the RF envelope is essentially rectangular. If the duration of the pulse (T) is extended to increase the signal energy, the bandwidth (hence range resolution) will be reduced. Modulation of the remaining variable, frequency (or phase), is the only means of expanding the bandwidth of the RF pulse. The resulting expression for the transmission is:

$$s(t) = \mathrm{rect}\left(\frac{t}{T}\right) \sin\left[\omega_0\, t + \varphi\,(t)\right],$$

where

$$\mathrm{rect}\left(\frac{t}{T}\right) = 1, \quad \frac{|t|}{|T|} \leq 1/2$$
$$= 0, \quad \frac{|t|}{|T|} > 1/2.$$

The choice of a particular modulating function $\varphi\,(t)$ is dictated by target characteristics and environment, the practicality of generation and reception (correlation) equipment, and the radar

Pulse compression is a well-known technique for improving radar performance. A method using binary 0°- 180° phase modulation waveforms with pseudo-random character has many attractive features which the advent of high-speed, low-cost microcircuits has made realizable. This article discusses some general properties of the modulating waveform and results obtained from a hardware implementation.

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function desired (search, track, etc). The output of a radar receiver is often presented to the observer as a two-dimensional plot, with amplitude as the ordinate and time as the abscissa. Target returns appear as pulses at a time corresponding to range, with amplitude and duration sufficiently different from noise to indicate a true signal. A three-dimensional plot of the receiver output versus target range and range rate is given by a surface called the ambiguity function. This function is the amplitude of the cross-correlation of signal return with the receiver filter, and shows the effect of doppler offset on time response. Woodward[1] is generally credited with applying this concept to radar. The width of the main peak of the ambiguity function (at half voltage points) along the range axis indicates the range resolution, and the width along the other axis gives the velocity resolution. A narrow pulse with good range resolution has relatively wide doppler response.

A widely used class of pulse compression is obtained by transmitting an RF pulse of constant amplitude in which the carrier is frequency modulated linearly, the frequency deviation ($\triangle F$) being many times the inverse of the pulse width.[2] Processing the signal returns in a filter with a linear time delay versus frequency characteristic of opposite sense to the linear frequency sweep of the transmission produces a compressed pulse output with an envelope approximating $\frac{\sin x}{x}$ having a width at the -4 db points of $\frac{1}{\triangle F}$. Filters using cascaded lattice networks are practical up to compression ratios of 30 to 40. For higher ratios (100 to 200), dispersive elements (having linear delay versus frequency) such as metal strips and quartz delay lines have been used. One objection to linear FM is the masking of small targets by the near-in time sidelobes of a larger target. For example, the first sidelobe occurring at $x = 3\pi/2$ is 13.5 db down from the central response at $x = 0$. Therefore the smaller of two targets separated by this amount will be obscured if the amplitudes differ by more than 13.5 db. This effect may be reduced by weighting the filter frequency response, with an attendant loss in sensitivity (≈ 1 db) and mainlobe broadening. Performance in uniform clutter approaches that of the ordinary rectangular pulse width of $\frac{1}{\triangle F}$, suffering a loss of only ≈ 0.5 db due to clutter contribution from the sidelobes. Linear FM is described as being "doppler invariant" in that a doppler offset of $\frac{1}{T}$ produces a time displacement of $\frac{1}{\triangle F}$ in the filter output with little amplitude reduction.

Another method of pulse compression involves the transmission of a set of contiguous pulses with equal amplitude and duration (τ), but which have different carrier frequencies. The signal returns thus generated are correlated using discrete delay lines and filters. Individual filter bandwidths are commensurate with the sub-pulse length τ, and the outputs of the delay lines are combined so as to provide coincidence between the subpulses. Choice of frequency order can be used to achieve doppler resolution if desired.

A third type of pulse compression is implemented

[1] P. M. Woodward, "Probability and Information Theory, with Application to Radar," McGraw-Hill Book Co. Inc., New York, 1953.

[2] J. R. Klauder, et al, "The Theory and Design of Chirp Radars," *Bell System Technical Journal No. 39, July* 1960, 745-808.

by modulation of the carrier phase in discrete steps. The phase modulation waveform is generated and applied to the transmitted RF, with the return signal phase correlated against the stored transmission waveform. A complete radar receiver employing maximum length binary sequences as the phase modulation (0°-180°) waveform has been constructed and tested at the Applied Physics Laboratory. Elspas[3] describes in detail the considerations of resolution in range and velocity together with ease of generation which make this type of signal attractive. Using digital techniques, the normal benefits of pulse compression can be obtained in a design that is essentially free of restrictions as to pulse length, carrier frequency, and compression ratio over a wide range of values. The unit described here has a maximum compression ratio of 255:1, with a nominal output pulse width of 0.1 μsec (10 Mc/s clock).

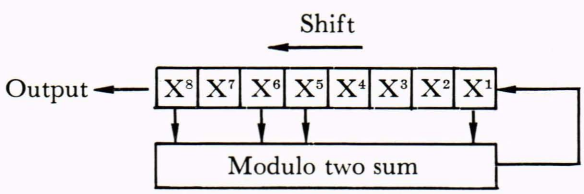
At this point, some discussion of the particular class of codes used and the respective properties of these codes is in order.

Maximum Length Codes

Binary pseudo-random codes can be generated by sensing predetermined stages of an "N" bit shift register and summing modulo two, with the result applied to the input of the shift register. The stages used in the feedback path correspond to the "1" coefficients of irreducible polynomials modulo two. In most tables, instead of using a binary representation, these coefficients are listed in octal notation. Thus, for example, for a degree 8 polynomial, length 255, written in binary notation 101100011, the octal notation is 543_8. The polynomial which this denotes is:

$$(1)X^8 + (0)X^7 + (1)X^6 + (1)X^5 + (0)X^4 + (0)X^3 + (0)X^2 + (1)X^1 + (1)X^0.$$

The coefficient of the X^0 term appears in every polynomial and refers to the closing of the loop onto the first bit in the shift register. A circuit diagram of a code generator for 543_8 (101100011) is shown below:



[3] Bernard Elspas, "A Radar System Based On Statistical Estimation and Resolution Considerations," *Technical Report No. 361-1*, August 1955, Applied Electronics Laboratory, Stanford University.

The following are some code characteristics:

1. All polynomials have an even number of "1" coefficients plus the constant 1 (shift register input).
2. If $2^N - 1$ is prime, all codes are maximum length.
3. If $2^N - 1$ is not prime, code lengths less than maximum also occur, and are factors of $2^N - 1$.
4. The initial condition in the register determines the cyclic permutation of the code.
5. The number of transitions in a maximum length code is 2^{N-1}.
6. The number of sequences K clock periods long (K "1's" or K "0's") between transitions is in general 2^{N-1-K}.

The use of binary codes (BC) for pulse compression radar requires knowledge of at least two properties of the autocorrelation function: (a) maximum values off the main peak (sidelobe level) and, (b) average power in the sidelobes (to compare performance in weather and other extended clutter environments). A computer program was written and data obtained on all BC's through degree 8 (255 length). The finite time autocorrelation function is formed by multiplying the code by a time-shifted replica of itself, and summing the product across the overlap. The autocorrelation function is even (symmetrical about $t = 0$) as usual, with $2(2^{N-1} - 1)$ sidelobes on each side of the main response. The initial condition (state of the binary elements) in the shift register determines the cyclic permutation of the code to be generated. Since there are 2^{N-1} initial conditions, there are 2^{N-1} different autocorrelation functions for each polynomial. The printout (for each starting point) consists of one side of the correlation function, the maximum sidelobe value and location, and the root mean square (RMS) of the sidelobe amplitudes. Initial condition is the decimal equivalent of the binary number in the register when code generation starts. A condensed list (Table I) shows the more interesting properties of the various codes (sidelobe values and RMS), and the corresponding initial condition.

The columns headed "Lowest Sidelobe" and "Highest Sidelobe" refer to the magnitude of the peak values of the autocorrelation function off the main response for each code, showing that there are "good" starting points (sidelobe maxima of 13 or 14) and "poor" starting points (sidelobes of 19

TABLE I

CORRELATION CHARACTERISTICS OF BINARY CODES

Degree (Length)	*Polynomial in Octal Representation*	*Lowest Sidelobe*	*Initial Condition*	*Highest Sidelobe*	*Initial Condition*	*Lowest RMS*	*Initial Condition*	*Highest RMS*	*Initial Condition*
1 (1)	3	0	1	0	1	0	1	0	1
2 (3)	7	1	1,2	2	3	.707	1,2	1.58	3
3 (7)	13	1	6	3	3,5	.707	6	1.78	3,5
4 (15)	23	3	Many (7)	5	5	1.39	2,8	2.25	9
5 (31)	45	4	5,6,26,29	6	Many (9)	1.89	6,25	2.87	7
5 (31)	75	4	Many (9)	7	7	1.74	31	2.87	25
5 (31)	67	4	2,16,20,26	7	6,27	1.96	6	2.92	15
6 (63)	103	6	Many (8)	11	5	2.62	32	3.87	51
6 (63)	133	6	Many (9)	10	47,49	2.81	35	3.94	57
6 (63)	147	6	Many (9)	10	31,61	2.38	7	3.92	45
7 (127)	203	9	1,54	15	9,42,86,106,112	4.03	109	5.31	44
7 (127)	211	9	9	13	Many (10)	3.90	38	5.26	109
7 (127)	217	8	33	15	75	4	33	5.26	79
7 (127)	235	9	49	14	40,46,85,125	4.09	12	5.14	40
7 (127)	247	9	104	13	Many (14)	4.23	24,104	5.21	73
7 (127)	253	10	54	16	49	4.17	36	5.2	57
7 (127)	277	10	14,20,73	15	77,91	4.15	50	5.31	5
7 (127)	313	9	99	13	Many (15)	4.04	113	5.39	95
7 (127)	357	9	15,50,78,90	14	11,51	4.18	122	5.15	34
8 (255)	435	13	67	19	Many (8)	5.97	135	7.06	221
8 (255)	453	14	Many (20)	21	114	5.98	254	7.16	44
8 (255)	455	14	124,190,236	22	114	6.10	246	7.15	66
8 (255)	515	14	54	21	51,56,197,227	6.08	218	7.10	161
8 (255)	537	13	90	19	29,195	5.91	90	7.08	63
8 (255)	543	14	Many (10)	20	71,91,203	6.02	197	7.10	202
8 (255)	607	14	Many (7)	20	51,241	6.02	15	7.1	48
8 (255)	717	14	124,249	21	59	5.92	156	7.2	127

to 22) for 255 length codes. These values are relative to 255 at $t = 0$. The computed sidelobes may have positive or negative values, but a receiver linearly detects the signal (takes the magnitude) and therefore the sidelobe magnitude is the important factor. It can be seen that for the best starting points the peak sidelobe level is less than the square root of the code length (or ≈ 16 for 255). Not shown in the table is the fact that the near-in sidelobes are even lower than those listed. The availability of a number of codes allows unsynchronized operation of similar radars without generation of spurious targets, since cross-correlation between different code sequences produces a noise-like output; however, coincidence of target return and transmission from the other radars will result in signal suppression.

For this type of signal a cut through the ambiguity function ($t = 0$) along the frequency axis shows the velocity (doppler) response to have a $\left|\frac{\sin \pi f T}{\pi f T}\right|$ shape, with the first null at $f = \frac{1}{T}$, where T = transmitted pulse length. This suggests that a proportional shift in the local oscillator could be employed to cause cancellation of ground clutter. The width of the compressed pulse determines range resolution.

Digital Receiver

The experimental digital pulse compressor (Fig. 1) consists of:

1. Code generator plus RF modulator
2. Search processor
3. Acquisition and track circuitry
4. Target simulator
5. Displays (range and velocity)

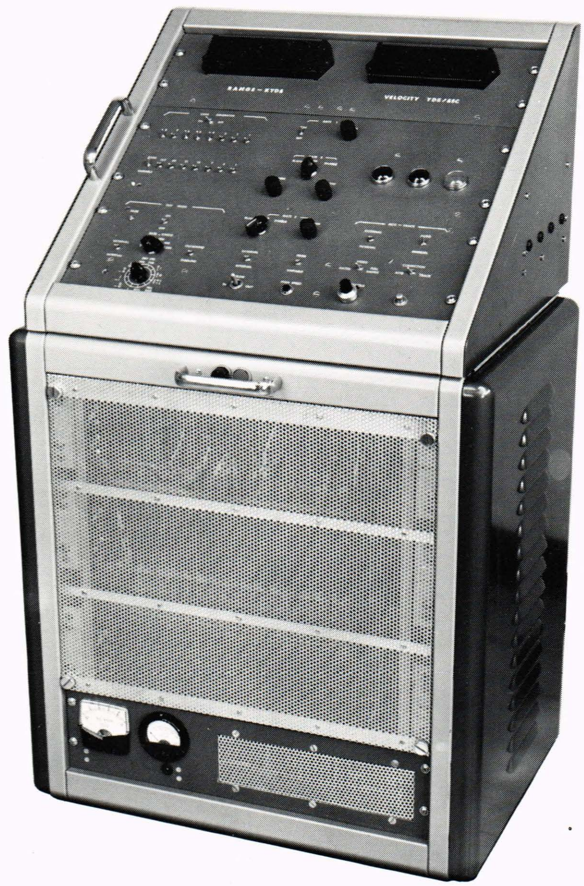


Fig. 1—255:1 digital pulse compression unit and range tracker.

Code generator operation is as described earlier. Options include variable pulse compression ($2^N - 1$, N variable from 8 to 1) from 255 down to 1. Less than maximum compression is preferred in some cases, as it reduces the number of range bins in a given interval. The starting point is variable from pulse to pulse, if so desired.

The code generator output is used to apply 0°-180° phase modulation to the transmitted RF pulse. This is most conveniently done at a low signal level prior to amplification up to the high power level of the transmitted pulse. Received target echo signals are typically converted to some intermediate frequency (e.g., 30 or 60 Mc/s) for amplification and filtering. Digital processing requires that the signals finally be converted to zero-frequency by mixing with a local oscillator at the exact IF frequency. Uncertainty in absolute phase which might result in loss of signal in this zero beating, or homodyne, process is avoided by mixing with a pair of quadrature local oscillators to yield two bipolar video signals, in-phase (I) and quadrature (Q). These I and Q video signals are limited and quantized (continuously at 10 Mc/s) as a "1" or "0" (+ or −) and applied to the signal registers.

The digital processor comprises the bulk of the equipment (Fig. 2), consisting of 32 identical cards (Fig. 3) with eight stages of correlation per card. These cards are connected in series, the I, Q and control shift registers each being 256 bits long, with two sets of logic gates (each 256) and two sets of summing resistors (I and Q) from the logic gate outputs. The code is shifted into the control register at a 10 Mc/s rate as it is applied to the RF modulator, setting up the correlation function for each transmission. Correlation is accomplished by summing the 256 bits of I (and the 256 bits of Q) such that the Kth I (or Q) bit's "1" or "0" side is used in the sum as dictated by the Kth control register bit. As the quantized I and Q videos are shifted down the signal registers, maximum output is achieved when the signal sequence matches the control register. The resulting compressed I and Q bipolar videos are linearly detected and combined in a "greatest of" detector ($|I|$ or $|Q|$ whichever is larger). This method of detection inflicts a loss of $\approx$ 0.46 db. The video thus generated is used for "A" scope (range) display and as search video to be compared to a threshold for automatic acquisition. Figure 4 shows this video output with a simulated injected target for varying amounts of pulse compression. With an uncompressed pulsewidth of 25.5 μsec, the output width at 255:1 ratio is 0.1 μsec. Long pulse outputs occur due to cycling of the starting point (all 0's producing no phase code). This results in the apparent dual traces in Fig. 4.

Automatic acquisition is achieved by providing a preset threshold over any given range interval. Search video is compared to this threshold, and the range tracker is positioned at the point where the threshold was exceeded. Subsequent returns from

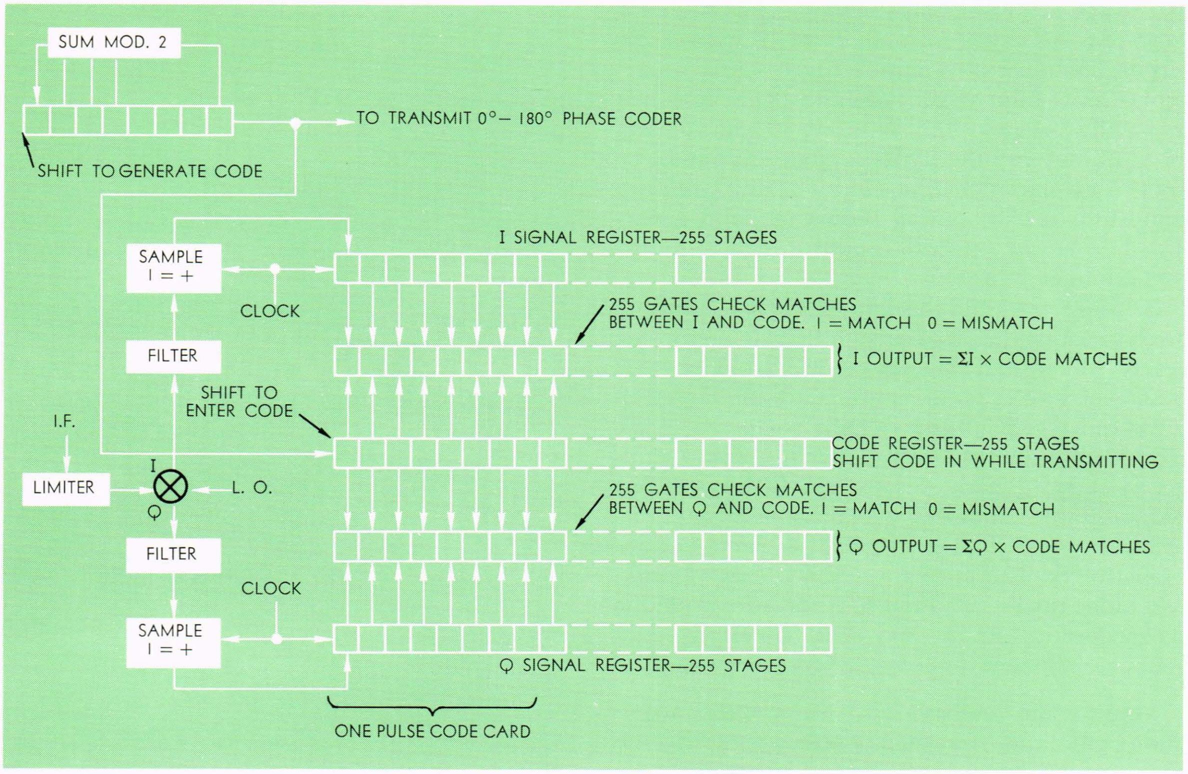


Fig. 2—Block diagram of digital compressor.

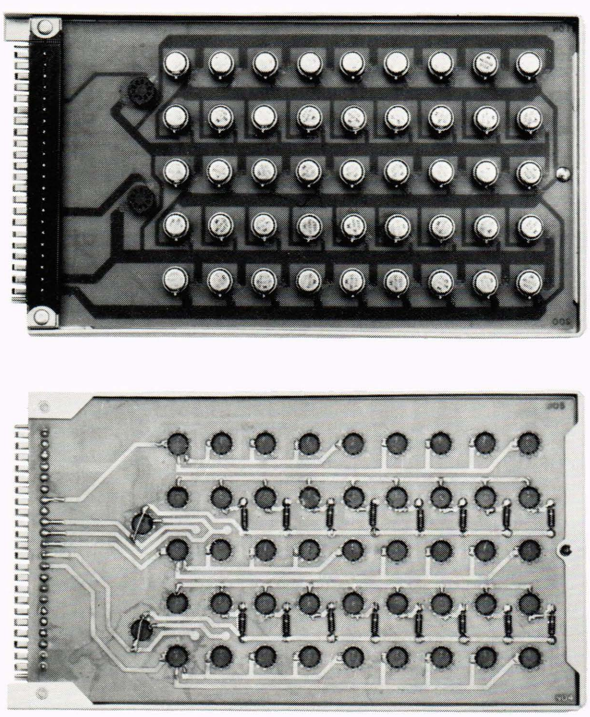


Fig. 3—Eight-stage digital compression card, four-layer board.

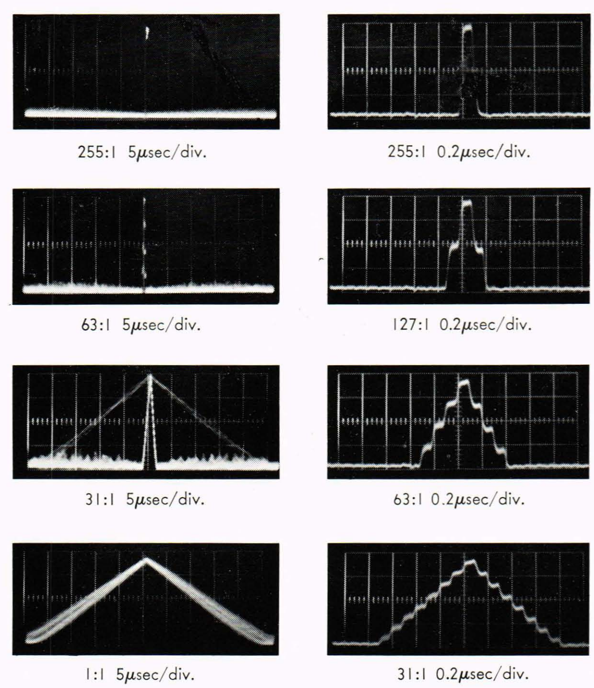


Fig. 4—Pulse compression outputs for varying amounts of pulse compression.

that range are examined for signal presence on a 3-out-of-8 hit criterion, and if this is passed the process continues to 8-out-of-16. To satisfy a particular set of false alarm rates and signal detection probability, ($P_{FA} = 10^{-6}$, $P_d = 0.9$, 10 kyd interval, 32 transmissions) the signal to noise ratio must be $\approx + 6$ db out of the detector. The tracking loop bandwidth is wide during acquisition, and is narrowed after an interval commensurate with expected target velocities.

In range tracking, the limited video (I and Q, pre-correlation) is separated into two paths, one (early) direct and one (late) delayed. The track code generator is set at the range of the target return, and the quantized video is compared to the code, with agreements accumulated in four counters (I early, Q early, I late, and Q late). These counters are compared and a range error signal obtained, as well as a measure of signal-to-noise ratio (S/N). The error is used to up-date the second order tracker, and the S/N measure is used in the acquisition, track, and coast circuitry. Track codes are also sent to the angle processing channels.

Thus far the emphasis has been on target acquisition and range tracking. There is another important function of a tracking radar employing a narrow beam antenna. This is the requirement to point the antenna so as to maintain maximum illumination on a target, just as a searchlight beam is aimed. While a detailed discussion of this radar angle track function would be too lengthy to include here, it is of some interest to consider how angle error processing might be handled for the digital pulse compression case. Figure 5 shows the essential elements of a so-called monopulse angle error processing system. The antenna is comprised of four radiating elements which, typically, share a common reflector or focusing element such as a parabolic dish. The feed elements thus produce four virtually identical antenna patterns which differ only in their pointing direction. With respect to the central axis of the antenna the pattern formed by feed element A is aimed up and left, that of B is aimed up and right, E is aimed down and left, and D is aimed down and right. Signals from the four feed elements are combined in the comparator, as indicated, to form a sum and two difference signals which are then passed through three gain and phase matched IF amplifiers, followed by matched filtering and phase detection to recover the pointing error signals, ϵ_{TV} and ϵ_{EL}. The form of matched filtering will depend upon the transmitted signal, e.g., for linear FM the filter will be a dispersive delay line. The point of interest is that three identical filters are required to preserve the amplitude and phase relationships between the sum and difference signals. For digital pulse compression the matched filter takes the form shown in the inset of Fig. 5. Multiplication by the track code, which is a delayed replica of the transmitted signal envelope, followed by narrow band filtering, results in a filter matched to the received signal envelope regardless of compression ratio. This arrangement requires substantially less equipment than would be required if three (two additional) shift register processors of the type employed for acquisition and range tracking were used.

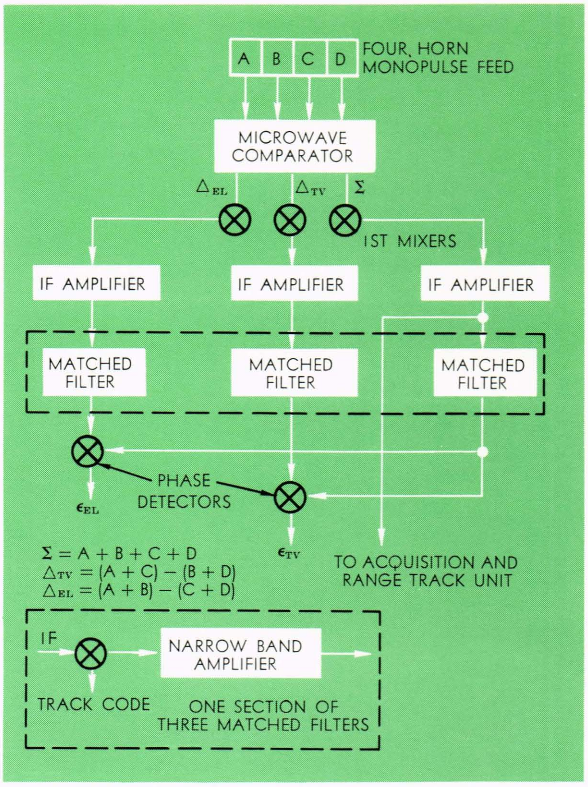


Fig. 5—Monopulse angle error processing system.

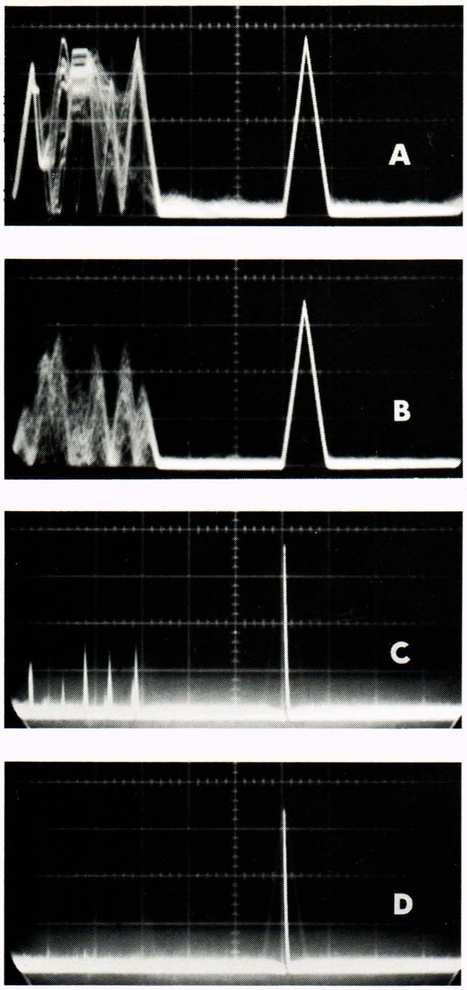
The target simulator consists of another code generator, RF modulator, and range rate controls. This portion of the equipment is used for sensitivity measurements and target injection in various environmental conditions.

The indicator devices are used to read-out range to acquisition gate (search), track gate (track), and target velocity. Range is read to ± 10 yards, and velocity to ± 1 yard per second.

Test Results

Figure 6 consists of A-scope (search) video photographs showing the effect of offsetting the local oscillator (and in some cases the injected target) by $\Delta F = \frac{1}{T}$ to achieve ground clutter nulling.

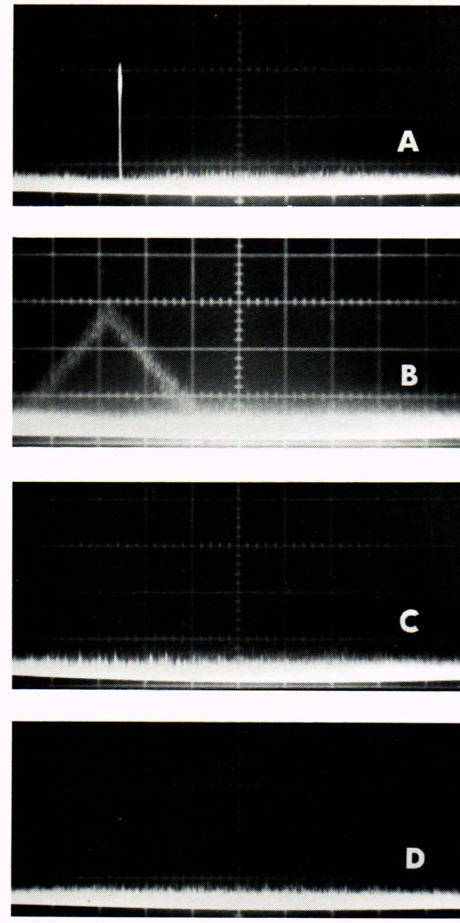
Search video output for various input codes is presented in Fig. 7. The low, noise-like outputs resulting from cross-correlating different codes illustrate freedom from mutual interference between like radars.



	CLUTTER POSITION RELATIVE TO RECEIVER VELOCITY RESPONSE	SIMULATED TARGET POSITION RELATIVE TO RECEIVER VELOCITY RESPONSE	COMPRESSION RATIO
A	PEAK	PEAK	1
B	NULL	PEAK	1
C	PEAK	PEAK	255
D	NULL	PEAK	255

SIMULATED TARGET RANGE = 6 (OR 6.5) CM
SIGNAL/NOISE RATIO ≈ 30 DB EQUIVALENT PREDETECTION
EL = 0.6°, AZ = 343.5°, 50 μSEC/CM.

Fig. 6—Photographs of ground clutter nulling with 255:1 digital pulse compression unit.



A RIGHT CODE AND STARTING POINT.
B RIGHT CODE STARTING POINT CYCLED THROUGH ALL VALUES.
C WRONG CODE, TWO TYPICAL STARTING POINTS.
D WRONG CODE, STARTING POINT CYCLED.

SWEEP SPEED = 10μsec/cm
S/N ≈ 20 db IN GREATEST OF $|I|$ OR $|Q|$

Fig. 7—Response of 255:1 digital pulse compressor to various codes.

Performance improvement in extended clutter (e.g., rain or snowfall) is illustrated in Figs. 8 and 9. There are two facets to consider when operating a radar in the presence of clutter: (a) the ability to establish a constant false-alarm rate (CFAR) and, (b) achieving an acceptable detection sensitivity. Figure 8 shows how the return from a rainstorm is progressively suppressed as the compression ratio is increased. The storm return is clearly visible in the center of the trace for the long pulse (no phase code) case. At a compression ratio of 255:1 the return is indistinguishable from the background noise level. Targets are detected by setting a threshold above the noise level and requiring that signals exceed this level to be recognized as targets. If noise alone, in the absence of a target, exceeds the threshold it constitutes a false alarm. The objective is to maintain the number of false alarms at an acceptably small figure while minimizing the prob-

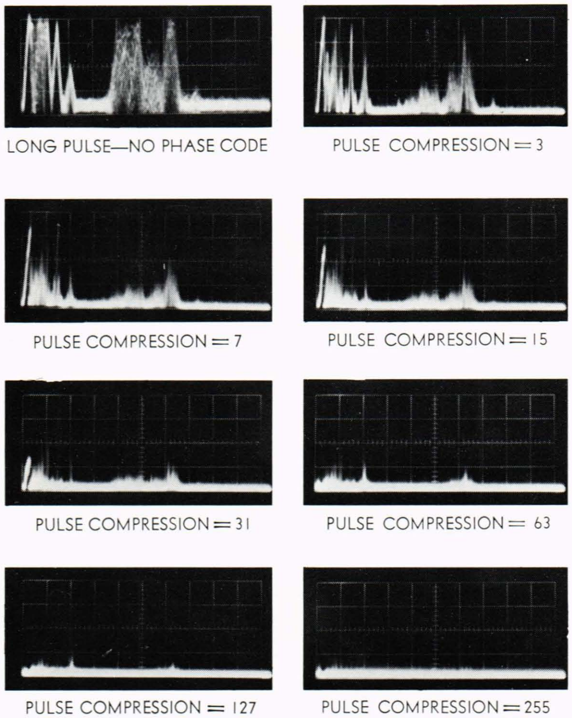


Fig. 8—Detected video out of digital pulse compressor on rainstorm (100 μsec/cm common to all).

ability that small targets will be overlooked. This process is facilitated if the background noise level is constant as for the 255:1 compression case. Clearly a fixed threshold set to exclude the rain return in the long pulse case would result in missing otherwise detectable targets occurring outside the interval of the rain. Figure 9 illustrates how pulse compression improves detectability in clutter. In

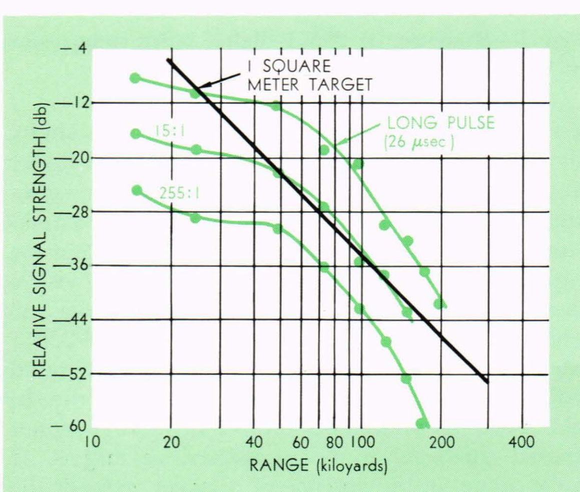


Fig. 9—Snow clutter profiles for three compression ratios. Condition dry snow (∽ 1 mm/hour water equivalent).

this case the clutter is a snow storm. The signal strength of the clutter return as a function of range is plotted for long pulse, 15:1 and 255:1 compression ratios, along with signal return from a point target of one square meter cross section. Since the signal return from the target does not vary as a function of compression ratio, the reduction in clutter return results in a direct improvement of signal-to-clutter ratio. The target would not be detected at ranges exceeding about 25 kyds for long pulse and 50 kyds for 15:1 compression, but would always be detectable with 255:1 compression.

Conclusions

The advantages of digital pulse compression are:

1. Simplicity of code generation and signal processing.
2. Flexibility—compression limited only by speed of microcircuits and length of transmission, variable compression ratio.
3. Signal integrity or uniqueness—availability of different codes, variation of cyclic permutation.
4. Standardization—multiple identical processing cards, independent of radar.
5. Constant false alarm rate action (CFAR).
6. Inherent freedom from adjustment of digital equipment.
7. Reliability.
8. Velocity resolution inversely proportional to transmitted pulse width.

The disadvantages of digital pulse compression are:

1. Limiting and quantization loss of $\approx$ 2db. (CFAR action normally costs 1 db).
2. Losses of $\approx$ 3 db in search video due to range cusping at maximum compression ratio (1 sample per code segment).
3. Wide transmission bandwidth $\left(\frac{\sin x}{x}\right)^2$ power spectrum with associated interference problems.

Acknowledgments

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