

The above line of argument can be extended indefinitely to show that if a shift of aim point is to be made after each shot, it should be by the amount $-x_i/i$, where x_i is the observed miss after shot No. i . Since the shift made at each stage is optimum, the whole operation of shifting by $-x_i/i$ after the i^{th} shot, where i is 1, 2, 3, . . . , etc., is overall the best way of aiming at a target when the results of the fire can be observed after each shot. The course suggested is optimal for a horizontal shift. Similar action for a vertical shift optimizes the aim vertically.

Since a shift of x_i/i horizontally and a shift of y_i/i vertically is equivalent to a shift of one i^{th} of the radial error toward the bull's-eye, we can write a very simple correction rule which is valid for two or more dimensions.

The Rule—After the i^{th} shot, where i is successively 1, 2, 3, . . . , shift the aim toward the target by one i^{th} of the observed miss (that is, by $1/i$ times the reported error).

It is of interest to consider the errors involved in following the above rule.

Suppose that the dispersion associated with each shot has a variance of σ^2 and that the error in aiming each shot has a variance of σ_o^2 . The variance of the reported position of a shot with respect to the actual aim point is thus $\sigma^2 + \sigma_o^2$ for each shot. When the rule is applied $n-1$ times, the correction applied amounts to shifting the aim point by the mean of the reported miss distances. The variance of this mean is $(\sigma^2 + \sigma_o^2)/(n-1)$, where $n \geq 2$.

The dispersion of the n^{th} shot has the single shot variance, namely σ^2 . The variance in the position of the n^{th} shot with respect to the bull's-eye is the sum of the variance of the mean and the single shot variance, or $\sigma^2 + (\sigma^2 + \sigma_o^2)/(n-1)$. Thus, as n increases, the variance in the position of the n^{th} shot about the bull's-eye tends to σ^2 . This is the variance due to the dis-

persion of the n^{th} shot. It is possible to reduce it only by improving the firing device.

Hitherto we have assumed that the variances of observations are constant from round to round. If the variances of observations do vary from round to round, as with some devices used for observing artillery fire, then the preceding rule can be generalized: After the n^{th} shot, shift on the x -axis towards the target by

$$\frac{1}{(\sigma^2 + \sigma_n^2) \sum_1^n \frac{1}{\sigma^2 + \sigma_i^2}} x_n,$$

where σ is the standard deviation of the fall of shot along the x -axis, σ_i is the standard deviation of the observation of the i^{th} impact, and x_n is the n^{th} reported miss distance. This rule is derived in a manner similar to that for the previous rule and is identical to it if σ_i is constant. A similar rule obtains for adjustment along another axis.

The distance between the target and the n^{th} impact along the horizontal axis is a random variable with mean zero and variance

$$\frac{1}{\sum_1^{n-1} \frac{1}{\sigma^2 + \sigma_i^2}} + \sigma^2.$$

The distances from the target along other axes obey the same statistical law, *mutatis mutandi*.

Conclusions

The optimum way to shift fire to take account of successive observed misses in aimed fire against a target has been given. The rule, as given, is a simple one. Since the rewards of using the optimum aim shifting procedure may be very great, it is surprising that no earlier mention of it has been found. It may very well be that good marksmen intuitively follow a rule close to that suggested.

APL COLLOQUIA

Oct. 15—"Ionospheric Disturbances Related to High Altitude Nuclear Explosions," by A. J. Zmuda, Applied Physics Laboratory.

Oct. 22—"Transport Phenomena in Gases," by C. F. Curtiss, University of Wisconsin.

Oct. 29—"Some Technical Problems of Arms Control," by L. S. Rodberg, U.S. Arms Control and Disarmament Agency.

Nov. 5—"Waves and Resonances in Solid State Plasmas," by S. J. Buchsbaum, Bell Telephone Laboratories.

Nov. 12—"Computer-Aided Patient Care in a General Hospital," by P. A. Castleman, Bolt, Beranek, and Newman, Inc.

Nov. 19—"Magnetohydrodynamic Generators," by A. Kantrowitz, Avco-Everett Research Laboratories.

ADDRESSES

- R. R. Newton, "Influence of Geodetic Uncertainties on Space and Ballistic Systems," *Tenth Symposium on Space and Ballistic Missile Technology*, U. S. Naval Training Center, San Diego, Aug. 4-6, 1965.
- J. R. Apel and A. M. Stone, "Experiments on Wave Interactions Between Plasma and an Electron Stream in a Magnetic Field," *Seventh International Conference on Phenomena in Ionized Gases*, Belgrade, Yugoslavia, Aug. 22-27, 1965.
- R. B. Kershner, "Present State and Further Development of Navigation by Doppler Measurements from Near Earth Satellites," *International Navigation Congress*, Munich, West Germany, Aug. 26-31, 1965.
- I. Katz, "Microwave Color of the Earth and the Moon," *I.E.E.E. 1965 International Antenna and Propagation Symposium*, Washington, D.C., Aug. 30-Sept. 1, 1965.
- The invited paper and the three discussion papers listed below were presented at the *Seventh International Symposium on Free Radicals*, Padua, Italy, Sept. 5-10, 1965:
- S. N. Foner and R. L. Hudson, "Mass Spectrometry of Free Radicals and Excited Molecules Produced by Electrical Discharges" (invited paper);
- F. J. Adrian, "Spin Lattice Relaxation as a Possible Complication in the Quantitative Determination of Gas Phase Hydrogen Atoms by Electron Spin Resonance";
- S. N. Foner, "On the Ionization Potential of the Methylene Free Radical by Mass Spectrometry";
- C. K. Jen, "Polarized Light Detection of Magnetic Resonance in Triplet Excited States."
- G. C. Weiffenbach, "Tropospheric and Ionospheric Propagation Effects on Satellite Radio-Doppler Geodesy," *International Association of Geodesy, Symposium on Electromagnetic Distance Measurement*,

Oxford University, England, Sept. 5-11, 1965.

- L. M. Spetner, "Information Transmission on Evolution," *1965 Joint Statistical Meetings*, Philadelphia, Sept. 8-11, 1965.
- S. D. Bruck, "Thermal Degradation of Piperazine Polyamides I: Poly(terephthaloyl trans-2, 5-dimethylpiperazine) and Poly(oxalyl trans-2, 5-dimethylpiperazine)" *American Chemical Society*, Atlantic City, Sept. 12-17, 1965.
- H. D. Black, "Attitude Determination Utilizing Redundant Sensors," *Sixteenth International Astronautical Congress*, Athens, Greece, Sept. 13-18, 1965.
- J. O. Engle and C. J. Nisson, "Simulation of Frequency Tracking Loops with Complex Targets," *Eastern Simulation Council Meeting*, APL, Howard County, Md., Sept. 16, 1965.
- F. F. Hiltz, "Simulation Techniques Used in Neuronal Studies," *Eastern Simulation Council Meeting*, APL, Howard County, Md., Sept. 16, 1965.
- F. F. Mobley and R. E. Fischell, "Orbital Results from Gravity-Gradient Stabilized Satellites," *Third All-Union Conference on Automatic Control*, Steamship Adm. Nakhimov, cruising the Black Sea, Sept. 20-26, 1965.
- S. D. Bruck, "The Pyrolytic Conversion of Poly(N,N'-p,p'-oxydiphenylene pyromellitimide) into a Semiconductor," *13th Canadian High Polymer Forum*, Ottawa, Canada, Sept. 22-24, 1965.
- R. E. Kemelhor, "Feasibility and Cost-Effectiveness Studies of a Naval Surface-to-Surface Missile System," First Lecture in a Series, *U. S. Naval Academy*, Annapolis, Oct. 14, 1965.

PATENTS

- E. A. Beck—*Radome With Particular Apex and Wall Structure*, Patent No. 3,195,138.
- E. L. Cochran—*Cavity Resonators*, Patent No. 3,205,432.

Editorial Board Change

Dr. R. E. Gibson, Director of the Applied Physics Laboratory, recently announced a change in the composition of the Editorial Board of the *APL Technical Digest*, as reflected on the inside cover of this issue. Dr. Donald J. Williams has retired from the Board, and Mr. Guy Worsley has been named to Board membership.

Dr. Williams, a member of the staff of the Physics Project of the Space Physics and Instrumentation Group, is a native of Jaffrey, New Hampshire. He received his B.S., M.S., and Ph.D. degrees in physics from Yale University, the latter in 1961, and held Silliman-Sloane, Bell Telephone Laboratories, and National Science Foundation Fellowships. From 1955-57, Dr. Williams served as a computer programmer in the USAF Air Research and Development Command. He came to APL in 1961 as a specialist in this field and in nuclear physics. He has been active in space research, especially radiation measurements, and was responsible for organizing the *Albedo Neutron Conference* held at this Laboratory in October 1963. He has served on the Editorial Board since 1963 and has contributed two papers to its pages: "The Earth's Albedo Neutron Flux," and "Studies of the Earth's Outer Radiation Zone." Dr. Williams is a member of the American Physical Society and the American Geophysical Union.

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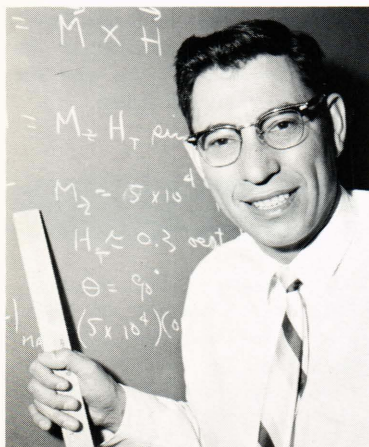
- M. H. Friedman (APL) and F. W. Churchill (University of Michigan), "The Absorption of Thermal Radiation by Fuel Droplets," *Chem. Eng. Prog. Symposium*, Ser. No. 57, **61**, 1965, 1-4.
- R. M. Fristrom, "Flame Structure and Flame Processes," *Chemical Engineer* (Birmingham University), **16**, 1965, 2-15.
- F. F. Hiltz, "Simulated Membrane Junctions and Additional Feedback Characteristics for an Artificial Neuron," *I.E.E.E. Trans. on Bio-Medical Engineering*, **BME-12**, Apr. 1965, 94-104.

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- F. W. Schenkel, "Rare Earth Oxide Dielectrics," *Proc. Fourth Annual Microelectronics Symposium*, Institute of Electrical and Electronic Engineers, May 1965, 2D-1-2D-6.
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- F. J. Adrian, E. L. Cochran, and V. A. Bowers, "Electron Spin Resonance Spectrum of FCO," *J. Chem. Phys.*, **43**, July 1965, 462-464.
- S. D. Bruck and H. E. Bair, "Determination of Number-average Molecular Weights of Polyamides in 90 per cent Formic Acid by Thermoelectric Differential Vapour Pressure Lowering," *Polymer—The Chemistry, Physics and Technology of High Polymers* (London) **6**, Aug. 1965, 447-450.
- R. P. Rich, "Is Computing a Profession?", *Computers and Automation*, **14**, Aug. 1965, 22-25.
- L. Monchick (APL), and E. A. Mason, R. J. Munn, and F. J. Smith (University of Maryland), "Transport Properties of Gaseous He³ and He⁴," *Phys. Rev.*, **139**, Aug. 16, 1965, A 1076-1082.
- C. R. Haave, A. J. Zmuda, and B. W. Shaw, "Very Low Frequency Phase Perturbations and the Soviet High-Altitude Nuclear Bursts of October 22 and 28, 1962. 1. Observations and Inferred Radiation Belts," *J. Geophys. Res.*, **70**, Sept. 1, 1965, 4191-4206.
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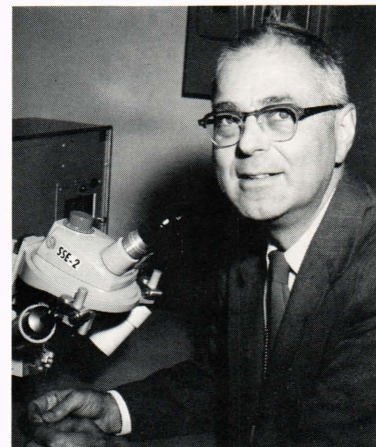
WITH THE AUTHORS



R. E. Fischell, author of "Spin Control for Earth Satellites," has contributed to the *Digest* on three previous occasions. Mr. Fischell holds an M.S. degree in physics from the University of Maryland and is a specialist in magnetics, satellite power and attitude control systems, and measurement and instrumentation. He is Supervisor of the Space Power

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R. C. Evans, author of "The APL Resistance Welder and Its Role in Satellite Reliability," is a native of Salisbury, Maryland. He received a B.S. degree in chemistry from George Washington University in 1938, and an A.M. degree in 1940 from the same university; in 1943 he received his Ph.D. degree in chemistry from The Johns Hopkins University. Dr. Evans came to APL in 1961 as a specialist in energy conversion, with many years' experience as a research



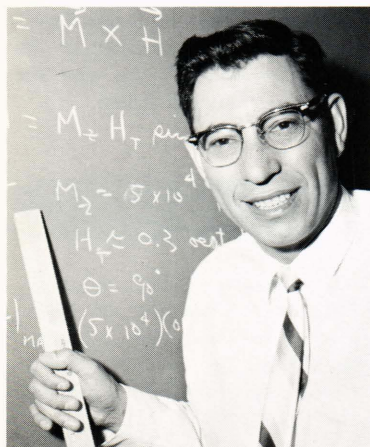
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