

*In the array of motions possible for a single drop or bubble moving through another fluid, it is interesting to examine the slow steady motion of small fluid bodies. Some experimental data are discussed in terms of a theoretical drag relation previously derived for low Reynolds number flows. The need for further research on the role of interfacial forces is indicated.*

## MOVING BUBBLES, DROPS, and other FLUID BLOBS

V. O'Brien

**L**ike many popular ideas, the "tear drop" shape is largely a misconception. Not that it can never occur, but by-and-large a drop of ordinary liquid moving in another fluid has some other shape. Moving bubbles and drops are often globular, as in the static case, but they may have diverse forms. The whole flow field associated with the motion of a fluid body occurs in such bewildering variety that a lifetime of study devoted to such flows would probably fail to explain them fully.

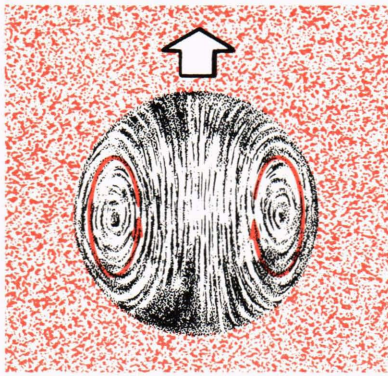
We shall concentrate here on the slow motion of small fluid bodies, a study that will point out both our knowledge and our ignorance. Such fluid behavior is an immediate practical concern of the chemical engineer in mixing and extraction processes; it may prove to be important to aerospace engineers in understanding the ablation of nose surfaces, in transpiration cooling, or in problems of hydraulic systems.

A bubble is a small mass of gas in a larger quantity of another fluid, either gas or liquid. Examples are the soap bubble and an air bubble trapped in tap water. A drop is a quantity of liquid in another fluid, either liquid or gas, as a water drop in oil or a raindrop in air. Usually one thinks of the bubble or drop fluid as being contained within a definite interface. However, the inner fluid may be completely miscible with the outer fluid and there may be no interface in

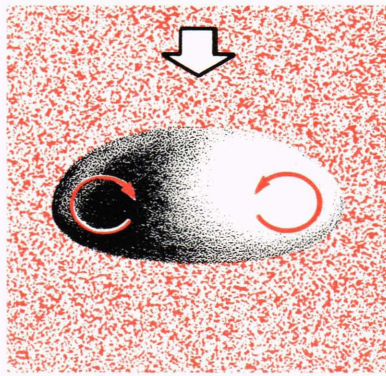
the usual sense, although there may be quite a sharp boundary between the fluids. The only word the dictionary has for this situation is *blob*, which includes *bubble* and *drop* as special cases.

There are specific reasons why a fluid blob moving in another fluid is more interesting to study than a solid moving in a fluid. A fluid blob can deform in shape and can flow internally. There are many types of motion possible because these characteristics are controlled by the viscous forces, the inertial pressure forces, and the interfacial forces in rather complex ways.

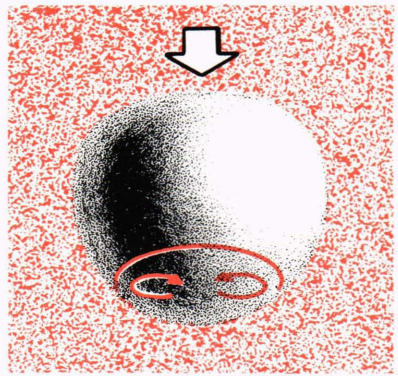
Consider a very simple set of experiments. Into each of several large quiescent reservoirs of the same fluid place a fluid blob having a density different from the others. According to the relative densities of the fluid pairs, the earth's gravity field will cause the blobs to move up or down. The shape of these moving blobs will be spherical, spheroidal, hemispherical cap, or too irregular for a simple description. The blobs may move vertically in a straight line, wobble back and forth in a vertical plane, spiral, or move irregularly. Inside the blobs there will be either no motion, partial circulation, or a full vortex. The over-all motion will be steady, have an unsteady periodic component, be partially irregular, or break up entirely. Depending on the choice of fluid pair and the size of the



**A**—SPHERICAL BUBBLE, STEADY STRAIGHT-LINE MOTION, FULL VORTEX.



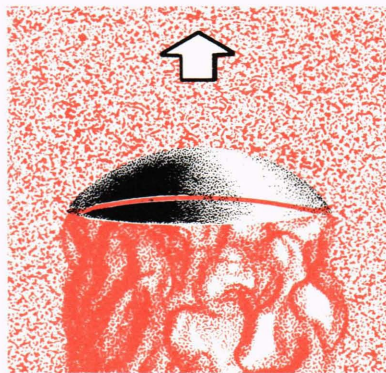
**B**—SPHEROIDAL FLAT-NOSED DROP, STEADY STRAIGHT-LINE MOTION, FULL VORTEX.



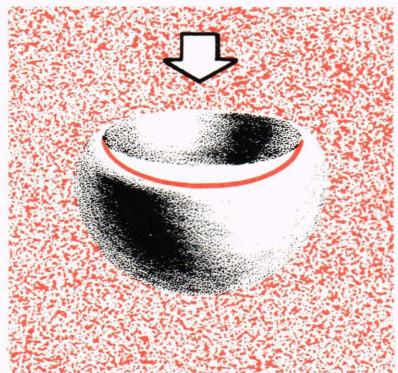
**C**—SPHEROIDAL FLAT-REAR DROP, STEADY STRAIGHT-LINE MOTION, PARTIAL CIRCULATION.



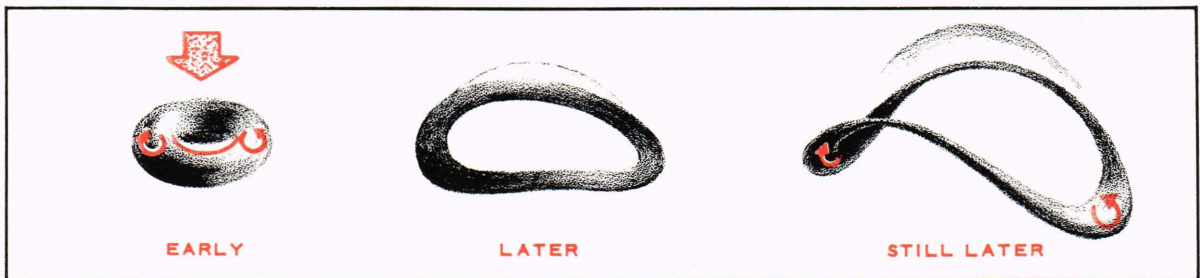
**D**—SPHEROIDAL BUBBLE, PERIODIC WOBBLE MOTION.



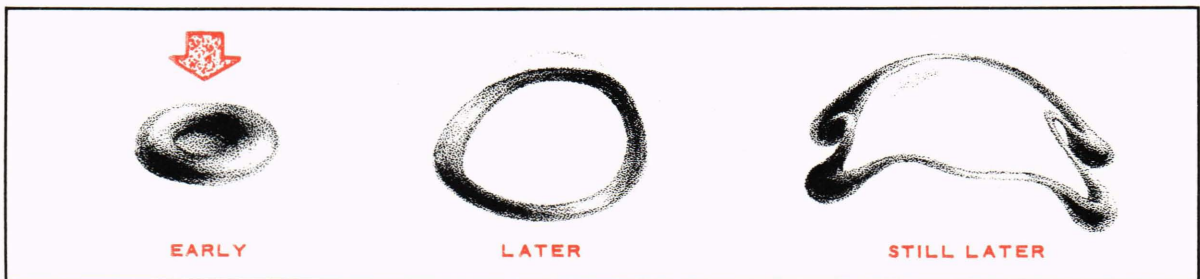
**E**—HEMISPHERICAL CAP BUBBLE, STEADY MOTION WITH TURBULENT WAKE.



**F**—DROP WITH INDENTED REAR, STEADY STRAIGHT-LINE MOTION, FULL CIRCULATION.



**G**—MISCIBLE DROP, UNSTEADY MOTION CALLED "VORTEX BREAK-UP."



**H**—IMMISCIBLE DROP, UNSTEADY MOTION TO BREAK-UP.

**Fig. 1**—A representative selection of qualitative flow patterns showing shapes and velocities of blobs, and vortex motion.

blob, there can be various combinations of the characteristics above. There can be one type of motion at one time and another later. Figure 1 illustrates qualitatively some of these cases, with arrows showing direction of blob motion. In general, the greater distortions in shape and unsteady motion are found for the larger blob volumes, except for the miscible cases that are unsteady for all volumes.

The initial conditions of the experiment consist of only two fluids and a blob volume. We might expect that such fluid fields could be explained quantitatively with a very few characteristic non-dimensional numbers; attempts to do this, however, have only provided qualitative information. It appears that not only the usual bulk fluid properties of viscosity and density, but the detailed conditions at the boundary between the fluids as well, are necessary.

## Bubbles

A comprehensive survey of single air-bubble motion in various fluids was published several years ago.<sup>1</sup> An attempt was made to relate the bubble drag coefficient—an easily-obtained net measure of the viscous retarding force—to the three non-dimensional parameters  $Re$ ,  $We$ , and  $M$ . Reynolds number,  $Re = Ul\rho/\mu$ , is the ratio of inertial and viscous forces; Weber's number,  $We = U^2l\rho/\sigma$ , is the ratio of inertial and surface tension forces; Rosenberg's system parameter,  $M = \rho\mu^4/g\sigma^3$ , is independent of bubble size and speed. In these relations,  $U$  is a characteristic velocity,  $l$  is a characteristic length,  $g$  is the gravitational acceleration constant,  $\sigma$  is the static surface tension, and  $\rho$  and  $\mu$  are the density and viscosity of the liquid, respectively. Universal curves in terms of these three parameters could not be drawn from the data since a complete description of the various flow regimes probably depends on still-unknown interfacial properties. It is possible, however, to relate the measured drag of the bubble quantitatively to certain features of the flow field. The three important features to be discussed are internal circulation, body shape, and Reynolds number.

For a spherical body and low  $Re$ , the governing fluid dynamic equation for steady motion has been solved. Flow patterns of the velocity field and theoretical drag values that compare

with measured drag values are found by varying the boundary conditions imposed at the interface. The Stokes flow solution ( $Re < 0.1$ ) for a spherical drop with complete circulation has been known for half a century. Surprisingly it does not depend on the static interfacial tension value  $\sigma$ . The tangential shearing stress is assumed to be continuous across the interface. The shearing motion of the outside fluid would in all likelihood cause full circulation of the viscous fluid inside. Thus, unless the interface has some resistance to the tangential shearing force, the fluid body is a spherical vortex.

The Stokes drag relation is an inverse linear function of  $Re$  for either a rigid sphere or a fully-circulating fluid sphere. The latter case depends on the ratio of the viscosities of the inner and the outer fluids. Typical variation is shown in Fig. 2 where the drag coefficient  $C_D$  for the fluid vortex sphere is less than for the corresponding rigid sphere. Stated another way, a circulating fluid sphere rises or falls faster than a rigid sphere of the same density moving in the same viscous liquid.

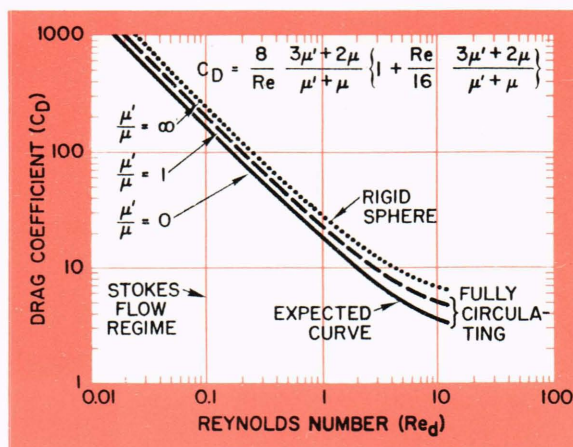


Fig. 2—Theoretical drag for spherical bodies.

It is possible for a fluid blob to retain its spherical shape beyond the low  $Re$  where the Stokes approximations are valid. Solutions correct to the first-order in  $Re$  show that the drag will be increased over the Stokes drag for both the rigid and circulating spheres (Fig. 2).<sup>2</sup>

Many blobs begin to deform to an approximate oblate spheroid even at relatively low  $Re$  (Figs. 1B and C). The Stokes drag for a solid

<sup>1</sup> W. L. Haberman and R. K. Morton, "An Experimental Investigation of the Drag and Shape of Air Bubbles Rising in Various Liquids," David Taylor Model Basin Rept. 802, Sept. 1953. (See also, Haberman and Morton, "An Experimental Study of Bubbles Moving in Liquids," *Trans. Am. Soc. Civil Engrs.*, 121, 1956, 227-252.)

<sup>2</sup> V. O'Brien, "An Investigation of Viscous Vortex Motion: The Vortex Ring Cascade," (Doctoral dissertation, The Johns Hopkins University, 1960).

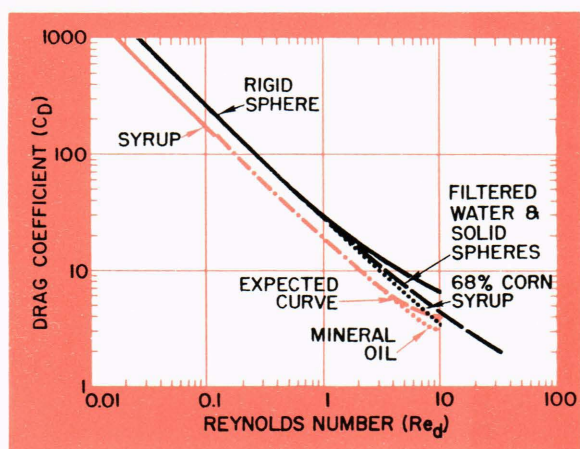


Fig. 3—Measured drag for air bubbles.

spheroid is well known and depends on the eccentricity of the meridian section. A flow solution for a spheroidal vortex has been obtained recently.<sup>3</sup> By matching tangential shear at the interface, a drag relation for a fully-circulating fluid spheroid can be obtained. It depends upon both the eccentricity of the cross-section and the ratio of blob viscosity to outside-liquid viscosity,  $\mu'/\mu$ , in opposing fashion. An oblate spheroid always has higher drag than a sphere of the same volume, drag increasing as the eccentricity increases. As with the fluid-vortex sphere, the circulating spheroid has less drag than the rigid body. The drag depends on the viscosity ratio, being least for vanishing  $\mu'/\mu$ . The effect is a translation upward of the three curves in Fig. 2 for a given spheroidal body.

Data for air bubbles rising in four typical liquids (from Ref. 1) are shown in Fig. 3. Without resistive, tangential, interfacial forces acting on these bubbles, one would expect full circulation. Because of the low gas viscosity, the data should lie on the lowest curve of Fig. 2 during the period when the shape is spherical. As  $Re$  increases, the shape is likely to become more spheroidal, with a resultant upward shift in drag (deformation depending on the resistance to the deforming normal pressure forces). At the very lowest  $Re$ , which is the Stokes flow regime, air bubbles seem to be rigid; the drag is just Stokes drag for a rigid sphere. Although Ref. 1 did not reveal circulation patterns within the bubbles directly, other work has shown that

they do exist, but only for the higher  $Re$ . At a certain  $Re$ , the circulation appears first at the front of the fluid parcel (Fig. 1C). As  $Re$  increases, the circulation covers a greater portion of the interior until the whole is circulating. It seems evident that the bubble in the syrup is circulating at very low  $Re$ ; the bubble in oil begins to circulate later and shows deformation effects at an  $Re$  of 1.0. Circulation within the bubble in either water or the corn syrup solution is inhibited throughout this low  $Re$  range.

There is no parameter yet to explain this difference in circulation. Without doubt, both circulation and deformation are intimately connected with the dynamic characteristics of the interface. Unfortunately, however, studies of these properties are spotty, and suitable data are very sparse.

## Drops

Data similar to those for bubbles have been presented for drops moving in liquids for a number of liquid pairs.<sup>4,5</sup> Unhappily, the overall pattern of behavior seems to have more variety than the gas bubbles at the higher Reynolds numbers. The differences are probably due to the range of interfacial tensions of these systems while the gas-liquid pair almost invariably has a high interfacial tension value. However, in the lower  $Re$  ranges the behavior is remarkably similar. Very tiny drops do not circulate at all, and measured drags lie on the Stokes rigid-sphere line. As  $Re$  increases, circulation begins inside the drops and the drag values drop from the rigid to the fully-circulating fluid-sphere curve. If deformation takes place, the drag curves rise above the sphere values.

For the fluid vortex sphere, the dip of the drag curve below the rigid-sphere curve depends on the viscosity ratio of the system. For comparison with air bubble data in Fig. 3, consider only the drop systems where the viscosity ratio  $\mu'/\mu \rightarrow 0$ . Figure 4 shows some data on tetrachloroethylene, bromobenzene, and ethylbromide drops through 80-100% glycerin, and tetrachloroethylene through a 68% corn syrup solution. These drops seem to be fully circulating at small  $Re$ . The validity of the first-order approximation theory is restricted to  $Re \leq 1$ , but apparently it is an overestimate for  $1 < Re \leq 10$ . Thus, the first-order drag rela-

<sup>3</sup> V. O'Brien, "Steady Spheroidal Vortices—More Exact Solutions to the Navier-Stokes Equation," *Quart. Appl. Math.*, **19**, 1961, 163-168.

<sup>4</sup> M. Warshay et al., "Ultimate Velocity of Drops in Stationary Liquid Media," *Can. J. Chem. Eng.*, **37**, 1959, 29-36.

<sup>5</sup> R. Satapathy and W. Smith, "The Motion of Single Immiscible Drops Through a Liquid," *J. Fluid Mech.*, **10**, 1961, 561-567.

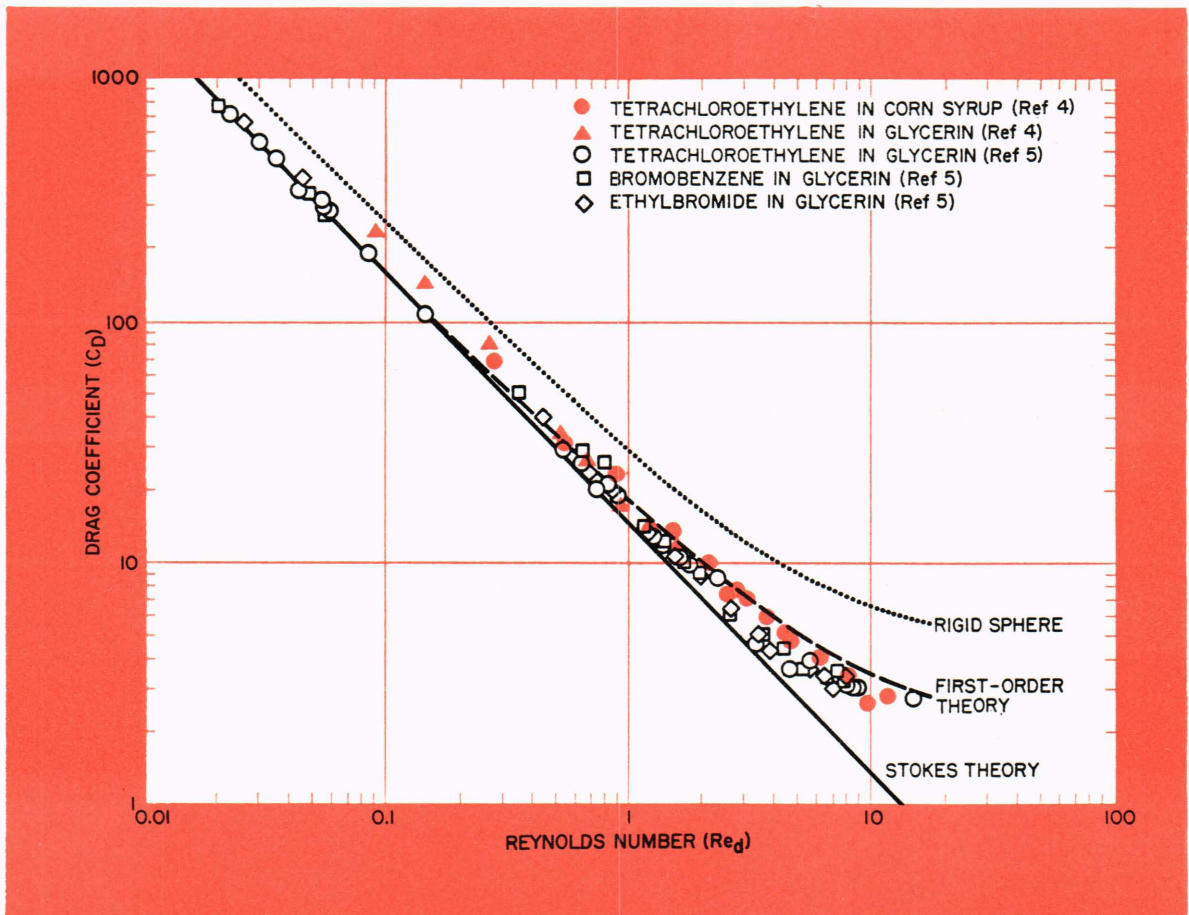


Fig. 4—Measured drag for falling spherical drops.

tion and the Stokes drag relation<sup>6</sup> provide upper and lower bounds on the actual drag in this range.

### Blobs

In the absence of an interface, the blob motion is always unsteady. The importance of the vortex pattern, and the change with time in appearance of a liquid blob until it ultimately breaks up (Fig. 1G), have been described before.<sup>7</sup> By increasing the drop volume sufficiently, in the case of immiscible falling drops, a point is often reached where the drop breaks up. The deformation and breakup for these drops are quite similar to the miscible blob behavior (see Fig. 1H and Ref. 7). It is very likely that the "vortex-breakup" behavior depends on the tangential, dynamic, interfacial characteristics.

<sup>6</sup> M. J. Hadamard, "Mouvement permanent lent d'une sphere liquide et visqueuse dans un liquid visqueux," *Compt. rend.*, 152, 1911, 1735-1738.

<sup>7</sup> V. O'Brien, "Why Raindrops Break Up—Vortex Instability," *J. Meteorol.*, 18, 1961, 549-552.

While no attempt has been made to cover fully all the aspects of the moving blob problem, the importance of the influence of the interfacial characteristics on the qualitative regimes of blob motion has been stressed. The main points of the low  $Re$  flow about bubbles, drops, and other blobs can be summarized:

1. Viscosity and shear cause circulation.
2. Circulation causes drag decrease (velocity rise).
3. Inertial effects cause drag rise.
4. Pressure forces cause change in body shape and, thus, further drag rise.
5. Interfacial forces oppose transmission of tangential shear and, therefore, oppose circulation.
6. Interfacial forces resist change in shape.

Much more research on the role of interfacial forces must be done before a quantitative explanation of the onset of circulation and the deformation of the blob can be obtained.